NEW AND OLD
METHODS OF ETHICS.

F. Y. EDGEWORTH.

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OR

"PHYSICAL ETHICS"

AND

"METHODS OF ETHICS."

BY

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BALLIOL COLLEGE, OXFORD;

BARRISTER-AT-LAW.

Oxford and London:

JAMES PARKER AND CO.

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THESE pages having been printed before the issue of the Second Edition of 'Methods of Ethics,' it has not seemed necessary to alter the references herein made to that work, as the doctrines referred to have not been substantially altered. The writer desires to withdraw all criticism on words which have been withdrawn from the 'Methods of Ethics.'

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I HAVE to thank my friend Mr. James Sully, author of "Sensation and Intuition," "Pessimism," &c., for having revised and corrected the following pages during their passage through the press, and for many suggestions.

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SECTION II.

NOTES.

PHYSICAL ETHICS.

IN the simple language of Locke, which it is easy to criticise and hard to improve, all knowledge is derived from Sensation and Reflection. Moral science is usually derived from Reflection; it has also been proposed to supplement Introspection by Physical Science. I attempt to compare these methods, examining a distinguished example of each, the "Physical Ethics" of Mr. Alfred Barratt, and the "Methods of Ethics" of Mr. Henry Sidgwick[1].

SECTION I.

The enquiry may be commenced upon the lines traced by Mr. Barratt in his attack on "Methods of Ethics," in "Mind," April, 1877. The principal question here at issue is, whether the immediate antecedent of all action is the prospect of pleasure, or of pain avoided, "desire" in the technical sense of the Mills'[2] "idea of pleasure in the future;" in the words of Mr. Barratt[3], whether "pleasures and pains are the only motives to voluntary action, and act in proportion to their intensity;" in the words of

[1] No doubt the line of demarcation between the two methods is very fine (as indeed it is between Sensation and Reflection). In so far as one employing the Introspective method appeals to the moral judgments, and generally the consciousness of *others* (e.g. Mr. Sidgwick's appeal to common sense), he must ascertain these phenomena of Reflection by physical marks; and in so far as these physical marks are ascertained by other physical marks (e.g. the hand-writing of a philosopher by print), he must take for granted and employ certain physical facts, propositions entirely made up of sensations. Whether Physical Ethic does more than this, may appear in the sequel.

[2] "Analysis of the Human Mind," by J. Mill. Note by J. S. Mill.

[3] "Mind," p. 173.

Mr. Sidgwick[1], quoting Mill, whether "we desire a thing in proportion as the idea of it is pleasant." Mr. Sidgwick is with Butler and Hutcheson, and a great company of English psychologists. Mr. Barratt is with Locke and the youthful Hume[2], and Mill, and Bain[3]. Mr. Barratt disputes the question on what he terms (I.) Physical, (II.) Introspective, (III.) Intuitional grounds.

I. The physical argument is to be gathered from the brilliant pages of "Physical Ethics." It is rather difficult to give the full force of the argument in an extract of moderate dimensions. The following is from our author's own "Conclusion."

"Moral science is a section of that division of Physics which treats of animate nature, and its special subject is the relation which exists between the active and passive elements of that nature. The fundamental principle, therefore, from which it starts, is the ultimate correlation of the two primary qualities of organized matter, irritability and contractility. The law of this connection is evidently derived from the laws of chemistry and electricity (taking these in the widest sense), but the exact particulars of its birth are at present unexplained, owing to the great imperfection of physiological science, and are quite irrelevant to the function of Ethics. * * This elementary law may be expressed as follows: Certain irritations of the tissue are followed by certain contractions of repulsion, and certain others by contractions which result in continuance of the irritation. The former are such as to injure or derange the tissue; the latter such as tend to its preservation and development. Animal motion is therefore at first merely the physical re-action of certain organized bodies upon others with which they come in contact. * * What objective science calls irritations, are presented to the rudimentary consciousness which we associate with the inferior animals, as merely a series of pleasurable and painful states, according as they tend to the preservation or injury of the organ respectively. The law of action, if translated into this phraseology, comes to be this, that a pleasurable state produces a reflex motion of acceptance, and a painful state, one of repulsion or

[1] "Methods," p. 30. [2] If Mr. Green's view be accepted. See Note C.
[3] But vide infra, p. 11, and Note G.

avoidance; all motion being comprehended under these two classes. Hence the physical correlation of contraction and irritation, when expressed in the inner language, comes to mean the aiming at pleasure and the avoidance of pain."

The reader, if he is not already a student of "Physical Ethics," should consult the whole passage, and is requested to read with particular attention the chapter on the Moral Sense, especially pp. 50—54, and the various recapitulations. Probably he will feel a new sympathy with the elemental powers of nature; he will admire familiar ideas adorned with eloquence and learning. When transport and admiration begin to cool into propositions, the following reflections may have been gained.

Of consciousness other than my own I know nothing, nothing but certain physical phenomena, such as are indelibly associated with my own consciousness[1]. The limits of this association are very arbitrary. Why associate consciousness with only certain movements of men and animals, why not also a rudimentary consciousness, pleasure and pain, with the simplest organisms and natural forces[2]? But of such forces of such simple organic movements are built up all, even the highest, human actions. All human actions take their rise in *pleasure* (to distinguish by italics *pleasure* as a mere physical phenomenon, a mode of matter and motion, from the conscious feeling of enjoyment; including in the term, absence of *pain*[3]).

[1] "Physical Ethics," pp. 238, 241, 290, &c. Compare Herbert Spencer, "Psychology," § 7, second edition.

[2] "The tissue acts so as to secure pleasure and avoid pain, by a law as truly physical and natural as that whereby a needle turns to the pole, or a tree to the light. Pleasure is only a name for the force which produces the second kind of movements (those securing continuance of an impression)."—Phys. Eth., p. 52.

[3] It is difficult to believe that no more valuable reflections in reference to the present question are afforded by "Physical Ethics." For in truth the value of a principle so very general as that presented in the text, may well appear doubtful. "Apparet nihil esse in illo quia omnia sunt." Had the assertion been not simply that pleasure is the cause of all movements, but that pleasure is the remote cause of all movements, for that all laws of motion have passed

A follower of Locke and Mill might go on. But the question at issue (supra, p. 1) concerns ideas of reflection, feelings of volition. What, then, can a proposition made up of sensations, a physical fact, contribute to the answer? "Nec nihil nec omnia." It can furnish a presumption, a ratiocination, to be verified by reflection. If the physical antecedents of my every action are made up of physical phenomena, such as are generally connected with the conscious feeling of pleasure, then there is a presumption that among the concomitants of every action will be that conscious feeling. But how slender the presumption appears from parallel cases. It might equally be argued that there was no such phenomenon as automatic action, at least in cases where the physiological conditions of consciousness and unconsciousness are not yet distinguishable. It might equally be argued that in mental chemistry the compound *is* the sum of the components, contrary to the best psychological und psychophysical[1] investigations. The cause of action, then, may be *pleasure* as a mode of motion, without being pleasure as a conscious feeling; which last *was affirmed by the negation of Mr. Sidgwick's doctrine.*

Mr. Barratt, indeed, does not speak very distinctly in the language of reflection. For him "Moral Science is a section of physical[2]." But if any one cares to take the longer route of *physical* terminology, there also can facts and Mr. Sidgwick confront him. Suppose all actions caused by *pleasure.* Still, it is important to note differences among movements. There is a difference between reflex actions and volitions proper (in Mr. Bain's sense of the term); a difference indicated by the different expressions of persons describing their feelings in each case, and further demarcated by physiological investigation.

through and grown out of a stage of (quasi) conscious appetition, are of the nature of habits, this proposition would be important, if true (infra, p. 13). But as the proposition so limited is not denied (in reference to human actions) by Mr. Sidgwick ("Methods," p. 42), so the limitation does not appear to have been made by Mr. Barratt.

[1] Mackintosh, J. S. Mill, Herbert Spencer, Helmholtz, § 60, &c.

[2] "Phys. Ethics," p. 288.

There is also (it is contended) a distinction between volition proper, hedonistic preference, and non-hedonistic preference; a distinction indicated by different sensible symbols—the *writing* of Aquinas, Butler, Hutcheson, Sidgwick, the *talk* of the plain man,—and capable of being further marked by physiology. In fact, some approach is made to this demarcation by the hypothesis of Mill[1], that the fixed idea (a species of non-hedonistic preference) is, like volition, a case of association; but not, like volition, of association with *pleasurable* phenomena. Even, then, for the morals which are a section of physics, the question at issue is an open question, to be decided by careful observation, not offhand by definition.

II. Next comes the proof by introspection, a legitimate appeal to internal experience; which, when Mr. Sidgwick legitimately makes, he should not even in a passing remark be described as a paralogist, begging "exactly what you have to prove[2]." Mr. Barratt, indeed, recognizes the principle that, "if a man feels a headache, he has one, and nobody can prove to him the contrary[3]." But in practice he is not perhaps so true as Mr. Sidgwick to the method of Locke. The reader should consult "Physical Ethics," part i. pp. 12—14, 17—22, and see whether, under a mass of axioms, there has not slipped in the proposition that non-hedonistic action is impossible.—The labours of the student are rewarded with a paper-money of definition, which after all may not be cashed at the bank of experience.

The case of "Video meliora proboque Deteriora sequor," Mr. Barratt meets with remarks such as Mill used to prove[4] that every action has a cause, but which are not perhaps equally useful, as a proof that there is but one cause of action, hedonistic desire.

"'Video meliora proboque Deteriora sequor.' In other words, action does not always follow knowledge. Of course not; but the doctrine does not require that it should, for it says, not that we follow what is our greatest possible pleasure, or what we know

[1] "Analysis of the Human Mind," note by J. S. Mill. Mill's hypothesis is psychological, but may well have a physical counterpart.
[2] "Mind," p. 177. [3] "Phys. Ethics," p. 29. [4] Exam.

or think to be so, but what at the moment of action is most desired[1]. In fact, the only practical measure of pleasures as motives at any moment, is in ourselves the resultant desire, in others the resultant action. But it may be objected, that to say that 'the pleasure which under any given circumstance is the greatest move,' and when asked for a measure, to say 'the pleasure which moves is under those circumstances the greatest,' is to argue in a circle. It is no more a circle, than to measure weights by their effect on the scales, or temperatures by the position of the mercury in a thermometer."

It is very difficult to seize the full force of this argument, owing to the uncertainty of at least one term. "Desire" seems to be used (the second time at least) as equivalent to the state of consciousness antecedent to, or concomitant with, action[2], rather than desire in the only sense which presents a significant issue[3]. So much, indeed, is perhaps to be granted to the physico-ethicist that it is very difficult to think, much more to communicate one's thoughts, about this subject, owing to the incurable obscurity of unaided reflection, and the corresponding indeterminateness of language: so that, perhaps, unanimity is not to be expected, until science has established (or proved to be impossible) a physiological distinction, as above suggested[4].

[1] See statement quoted above, p. 1.

[2] Compare "Methods," p. 33. "If by pleasant we mean that which influences choice, exercises a certain attractive force on the will, it is not a psychological truth, but a tautological assertion, to say that we desire what is pleasant, or even that we desire a thing in proportion as it appears pleasant."

[3] Suppose the question to be, whether all substances expand as their temperature is increased. Suppose a careful observer to have pointed out comparatively rare instances, like water not expanding as its temperature is raised to 39°. What should we think of the following sort of objection: 'In other words, expansion does not always follow the phenomena of (?) increased temperature, of course not; but the (right) doctrine does not require that it should, for it says that expansion follows, not the phenomena of increased temperature, but the (?) diminution of density. In fact, the only practical measure of the agency of temperature is, the resultant diminution of density, or the expansion. It may be objected that to say that the temperature, which is in any circumstances the greatest, expands, and when asked for a measure to say, the temperature which expands is under those circumstances the greatest, is to argue in a circle. It is not a circle to measure temperatures by the position of mercury in a barometer.'

[4] Supra, p. 5.

Still, remarking on the case of *ἀκρασία*, Mr. Barratt supposes the force of motive, ideas of pleasure in the future, to diminish, like the attraction of bodies, with the distance—in the inverse ratio of the square of the distance in time[1]! And again, "Action looks at life as we look at a landscape." But, as Leibnitz[2] observes of Locke's comparison of distant pleasures to distant bodies appearing small, there is only a partial propriety in it. As Mr. Green acutely remarks, criticising Locke[3]: "In whatever sense it is true of present pleasure and pain that it really is just as it appears, it is equally true of the future." Probably[4] Leibnitz has given the most satisfactory explanation of the mystery of *ἀκρασία*[5]. The knowledge of the better things seen and approved is only blind, "sourde," symbolical; the knowledge of the worse things followed is intuitive (the terms being used in Leibnitz' sense, so well explained by Sir W. Hamilton). So, then, when a man (to take Locke's instance) prefers the pleasures of a soaking club to the joys of heaven, his belief in the latter is merely a sort of $x = y$. But that the degree in which the conception of pleasures become symbolical, is proportional to, or any simple function of, the distance in time, may well be doubted. With the same religious doctrines a person is very differently affected at different times, by the prospect of rewards and punishments after death. The probable distance of the event seems by no means the most important factor in the vividness of the conception[6].

Mr. Barratt seems to admit[7] (what Mill admits in "Utilitarianism") that habit is an exception to, a modification of, the general hedonistic rule. This suggests an eirenicon. Non-hedonistic preference is ancestral habit[8].

[1] "Mind," p. 174. [2] *Nouveaux Essais*, ad Locke, c. 21, § 63.
[3] "Introduction to Hume's Moral Philosophy," part ii. See Note A.
[4] *Nouveaux Essaies.*
[5] "The dread mysterious hour
Of the tempter's subtle power."
[6] The juvenile Hume is less suggestive than usual on this question. Treatise, Part III., §§ vii. and viii. [7] "Mind," p. 175. [8] Infra, p. 11.

III. Next comes the intuitional proof. "Intuitionism," under the "Philosophic" treatment of Mr. Sidgwick[1], yields two arch-principles, Equity and Benevolence, which do not recommend themselves to Mr. Barratt. "Nothing," said Mr. Sidgwick, "can be right for me which is not right for all persons in similar circumstances." On this Mr. Barratt comments:—

"Now, if under circumstances he includes *internal* circumstances, such as character and belief, his hypothesis is self-contradictory, because different beliefs as to what is right are different circumstances; if not, the conclusion is false; for common morality says, that a man ought to act not only according to his belief, but according to the whole of his nature; and that what is right for one man may be wrong for another. The only fundamental assumption either necessary for a science of Ethics, or warranted by common notions, is that morality conforms to the general law of uniformity, i.e. that in the same circumstances, external and internal, the same thing is morally good[2]."

"I have already tried to shew, that it is either an assertion that morality follows the physical law of uniformity (i.e. that mere difference of individuality in moral agents, as in atoms, does not affect the result, which is precisely *similar* under all similar conditions), in which sense I gladly accept it as a testimony from consciousness to the possibility of a physical science of Ethics, or if 'the like case' does not include the like internal natures of agent and recipient, that it is not only no axiom, but plainly repugnant to common sense. Mr. Sidgwick, if I understand him rightly, takes it in the latter sense, yet holds it as an axiom[3]," &c.

Here, then, we are told, if the term circumstances be distributed, the statement is a truism re-asserting the law of causation; a truism which is inapplicable to the case of two men acting in different ways, and each thinking himself right; "because different beliefs as to what is right are different circumstances." If by "circumstances," he meant some circumstances, the statement is false.

But similar objections might be made to the proposition, "Mathematical judgments are the same for all persons, the data being the same;" which yet expresses an important

[1] "Methods," Book iii. ch. 13. [2] "Mind," p. 170. [3] Ibid., p. 180.

distinction between geometry and, say, gastronomy[1]. In the former, the conditions under which unanimous judgments are formed are generally definite; in the latter, they consist of (as yet) unknown dispositions of nerve fibrils.

As to the second principle of philosophic intuitionism, Mr. Sidgwick might deserve to have his logic unmercifully chopped up, if he could be suspected of trying to make utilitarianism out of egoism by a "logical jugglery[2]." But surely that high logic was addressed not to the pure egoist, but to one who has already one foot on intuitional ground. Such a one the philosopher addressing, endeavours to persuade him that, if he does not scatter against, he must gather with, the utilitarian. The pure egoist is clearly conceived as not amenable to utilitarian proof[3], and indeed, when it is reasoned[4] that (to use Mr. Sidgwick's apt terms) where the ultimate reason is egoism, the method cannot be utilitarianism, the case may have been overstated[5]. It is to be regretted that the author of "Physical Ethics," in criticising Mr. Sidgwick, should have adopted the tone of those who exhibit Mill's proof of utilitarianism as a logical fallacy.

Upon the whole, it may appear that Mr. Barratt's arguments, however valuable for those who have already embraced the narrow creed of an egoism[6], not only pure but doubly-distilled (which neither in calm, nor even in excited, moments acknowledges any spring of action but

[1] The illustration suggests a fanciful addition to the Sidgwickian principle. Consistently with quasi-mathematical certainty, there may be more than one thing-which-ought-to-be-done, as there may be several roots to an equation. In fact, this is the common case of indifferent actions (e.g. those which are supposed equally to tend to the general good). The reply is, that only one *method* can be right. There may be any number of roots, but only one equation. Suppose, however, the form of the equation given by a *functional equation* admitting of several solutions; suppose two *methods*, universalistic and egoistic hedonism, each to satisfy the (unconsciously implicit) condition of rightness, still the condition would be *single*. The first Sidgwickian axiom would survive, even if the second should fail in the manner indicated.

[2] "Mind," pp. 181, 184. See Mr. Bradley's pamphlet, p. 36, &c.

[3] Cf. "Methods," p. 461. [4] Book ii. and iv. [5] Infra, sect. 2, v.

[6] See note B.

the pleasure of the agent), however edifying to the converted, have not much controversial value as against the moralist, who, finding in his consciousness intuitions of beauty and truth and duty and the good of others, is determined to pursue those objects rather than the subjective feeling of pleasure. For the intuitivist the physical argument wielded by our author, however splendid, is rather of the nature of *σοφία* than *φρόνησις; περιττὰ μὲν καὶ θαυμαστὰ, ἄχρηστα δὲ.*

This view might be strengthened, and the whole subject illustrated, if it were considered on what suppositions Physical Science, by any means other than the ancillary functions which would be generally allowed to her[1], could contribute to the settlement of the great questions agitated by our authors. A preliminary postulate is the interdependence of body and mind, that to every phenomenon of reflection are conjoined certain phenomena of sensation: so that definite physical phenomena (which Mr. Barratt, if he please, and if he is consistent, may call *pleasure,*) are the cause of all human action; the cause also, or the Baconian form, of pleasure as a conscious feeling, the end of the pure egoist, and not only of that state of consciousness, but of every species of non-hedonistic preference (if such there be), of various states associated with various names, *ἀγαθὸν, ἐνεργεία,* duty, practical reason, moral sense, the affection of Butler, the sentiment of Hume[2], the reason-constituted[3] desire of the Hegelian, and that fixed idea, which supports the sympathy which crowns the edifice of Bain's utilitarianism. Let this be taken for granted on the authority of innumerable savants[4].

[1] Supra, p. 1. [2] See Note C. [3] See Note C.

[4] "I hardly imagine there exists a profound scientific thinker, who has reflected upon the subject, unwilling to admit the extreme probability of the hypothesis that, for every fact of consciousness, whether in the domain of sense, of thought, or of emotion, a definite molecular condition of motion or structure is set up in the brain."—Tyndal, "Scientific Materialism."

"Few, if any, will now deny, that with each display of mental power there are correlative changes in the material substratum."—Maudesley. Compare Fechner, Wundt, &c.

Let it further be supposed, I. That all actions have, as more or less remote antecedent, experienced pleasure; experienced, either by the agent himself, or by his ancestry. Let us consider separately (*α.*) the evidence, (*β.*) the ethical importance, of this proposition. The evidence (for the less obvious and only disputable part of the proposition) is none other than the conclusive evidence given by Herbert Spencer in favour of evolution, especially in his chapters on the feelings and the will. ("Psychology," 2nd edition[1].) The principal examples of alleged non-hedonistic preference will sustain this opinion. According to Butler, the objects of "affection" are not so much pleasure, as something external. Expectation of pleasure in consequence of experience is a case of self-love, not of those particular affections which rest in their ends. The distinction made by Butler has been represented by Bain as a mere juggle; yet it is supported by the highest names in Moral Philosophy, by Hume and Hutcheson and (may we add?) by Sidgwick. Is it not possible that both Bain and Butler are right, that the distinction noticed by Butler exists, and yet that all affections are generated by association with experienced pleasure—only that the association is mainly *ancestral* in the case of "affections" proper? The dim remembrance of ancestral pleasures, the force of ancestral habit, produces that propension of which Butler speaks, disproportionate to (distinct) expectation and (personal) experience of pleasure. It is remarkable that Mr. Bain, the almost contemptuous critic[2] of Butler, should himself start a non-hedonistic impulse, which appears in some ways considerably more antagonistic to volition proper than the Butlerian affection, which puts us upon seeking even what is *painful*, leads us to throw ourselves down precipices, &c.[3] This is the "fixed idea,"

[1] It does not appear that Mr. Spencer had occasion to consider specially the controversy concerning non-hedonistic action; but it seems to follow from his principles, that in so far as actions have become organized, they are performed *out of proportion* to the (present) pleasantness of the idea.

[2] "Mental and Moral Science," sub voce Butler.

[3] Ibid., sub voce Fixed Idea, passim.

which operates not by way of volition proper (what has been called above desire[1], in a technical sense), but operates throughout the whole range of passions; and, though styled "morbid" and "irrational," would seem to be both natural and indispensable, if sympathy is a case of the fixed idea. To take this conspicuous example, is it not probable that sympathetic action also is a case of association with pleasure, with the common pleasures of a remote ancestry, in the manner traced by Herbert Spencer[2]? As to the passions generally, perhaps the same account may be given of the fixed idea as about the Butlerian affection; and, indeed, it seems to exceed the powers of the microscope of consciousness (already sufficiently strained to perceive the existence of the genus non-hedonistic action) to distinguish the species. And as to the persons with fixed ideas leading them to throw themselves down precipices and otherwise "abimer" themselves, the world, as Mandeville would say, yet never abounded with them. So, again, as to reason, which has been spoken of by many philosophers as originating action. The observations of Herbert Spencer[3] on *à priori* propositions generally, *mutatis mutandis* and taken in connection with his chapters on feeling and volition, render it probable that the *à priori* imperatives of the practical reason are of the nature of ancestral habits. The criticisms on Kant[4] in "Physical Ethics," require only a change of *tense* to harmonize with this opinion. And generally, as the conclusion here reached seems chiefly to differ[5] from Mr. Barratt's in assuming conscious pleasure, not as an immediate, but remote, cause of action, so it may find a lucid exposition of premisses in "Physical Ethics."

(*β.*) The principle thus evidenced may be applied to

[1] Supra, p. 1. [2] "Psychology," part viii. chap. 5. [3] Ibid., § 208.

[4] "It is evident that a man can will to follow a law only by liking and choosing to follow it, or in other words, only because he is inclined so to do. Law, therefore, can act only through inclination, and hence such a notion of duty as Kant proposes is void."—Phys. Eth., p. 193.

[5] The difference might not have been suspected, but for the author's own comment in the article in "Mind." It cannot be ignored, after the author's contradiction of Mr. Sidgwick's doctrine.

practical Ethics in more ways than one. (1.) It has been asserted that ancestral habits are the best criteria of the useful,—to indicate briefly a doctrine which largely pervades current literature, and which is stated and estimated by Mr. Sidgwick[1] with his usual clearness. To offer any confirmation of his estimate would hardly be pertinent here; but it is not irrelevant to remark, in reply to Mr. Barratt[2], that the case is not quite parallel to geometrical axioms, most certain because confirmed by the most primitive experience. It is rather as if the geometrical circumstances of our ancestors had differed from ours; so that our inheritance of axiom should relate to those worlds, of which Professor Clifford tells us, where the shortest line is *not* a straight line. (2.) Next may be noticed, in order to be rejected, a deduction which, perhaps, was never seriously put forward by any one; that because the earlier stage in the genesis of action consists of conscious pleasure-seeking (according to the hypothesis here entertained), and because "the earlier condition of our impulses is somehow better and more trustworthy than the later[3]," therefore that egoistic action is somehow better, and to be exclusively pursued. (3.) But, though the preceding considerations be estimated at their right value by Mr. Sidgwick, the proposition under consideration may not be quite so otiose as he supposes. Even for the intuitivist moral imperative is bounded by physical possibility; as Butler, for instance, assumes in interpreting the precept to love our neighbour as ourself. It is vain, then, to recommend a course of action, not tending to the pleasure of the agent, as right, reasonable, &c., *if it can be shewn* that never in any past stage of evolution could such actions have tended to the pleasure of the agent sufficiently to produce an ancestral habit. Here, then, is a real negative criterion, no doubt involving a condition, but one which may be satisfied by history. By this criterion the "ascetic systems" denounced by Bentham may (when found at all) be found wanting. It is more important to remark, that pure utilitarianism appears not to sustain this criterion.

[1] "Methods," p. 432. [2] "Mind," p. 170. [3] "Methods," p. 42.

If the absolute self-devotion required by pure utilitarianism does not (as Mr. Sidgwick thinks) now tend to the pleasure of the agent, much less has it so tended at any previous stage of society.

II. A second supposition favourable to Physical Ethics is, that physiology could discover the cerebral cause of non-hedonistic action generally, and of that particular kind of non-hedonistic action which is directed to the pleasures of others (corresponding to sympathy in Mr. Bain's sense). Considering that the cerebral conditions of normal hedonistic action, and also of some reflex cerebral movements, are in a general way ascertained, it cannot appear absurd that the conditions of non-hedonistic action also (if such there be) should be ascertainable; especially if the latter kind of action should be, as implied in the preceding supposition (I.), (*quâ* non-hedonistic) a sort of higher species of reflex action. And the particular species of non-hedonistic action which aims at the pleasures of others, might so far resemble the known case of hedonistic action, as to be particularly amenable to physical investigation[1].

And here arises the nice question, whether physiology could ever be in a position to assert that there is no such process as non-hedonistic action. On the principles already employed it seems that, if I vividly and distinctly feel an ache, or the absence of pleasure, the authority of all the savants in Europe concluding the contrary could only produce surprise, not conviction. They might shew grounds, indeed, to suppose that I had experienced a pleasure, and forgotten it (just as Stewart supposed the links of association often to have been present in consciousness, and forgotten),—a supposition which does not seem pertinent to the present case. But if I had an obscure consciousness as to the *amount* of pleasure experienced (a supposition not irrelevant), then I should be disposed to accept the verdict of physical science. If the highest au-

[1] The peculiar relations which this particular species of non-hedonistic action bears to volition proper, and its consequent advantage over other species, were pointed out to the present writer by Mr. Sully. Supra, p. 5.

thorities persisted in bearing witness to a distinction apprehended by reflection, of which no physical indication could be traced, I must suppose that physical investigation has not been pushed far enough.

The consequences of such a discovery as that here supposed would be important. Doubly-distilled egoism[1] would learn, that there are more things in human nature than its philosophy had dreamt of. He who had hitherto believed no action possible, unless for the sake of (proportionate) pleasure, would now be affected as we inhabitants of earth might be upon observing in Venus a race with a magnetic sense—a mechanism related to magnetism, as the eye to light. Generally, the pure egoist would be put upon cultivating the sympathetic species of non-hedonistic action; *if* it could be shewn physically (what introspection renders probable) that he who stunts the sympathetic impulse, impairing thereby some delicate process of his organism, loses the fine flower of happiness,

> "shrivels up an empty soul,
> And reaps the wounds and wrath of wrongèd law[2]."

Especially might this be proved, if we could verify physically the tendency of a single action to destroy a habit[3], (a tendency, by the way, of first-rate importance in the utilitarian theory of exception to general rules).

To the intuitivist, *quâ* prudent, the preceding considerations equally apply. Moreover it may be expected that the investigation of the physical conditions of non-hedonistic preference generally, and specially of sympathy, may bring some support to a rather exposed point in Mr. Sidgwick's system; where he may be confronted not only by Plato and Jowett[4], but by his own admissions;—the position, namely, that while non-hedonistic pursuit is natural, rational, and indeed necessary to the existence of the intuitive method, yet that, ultimately and intrinsically,

[1] Supra, p. 9.

[2] Prof. G. F. Armstrong, "Tragedy of Israel," David.

[3] Mr. Bain has some striking remarks on the importance of avoiding defeats in the training of the will.

[4] Plato Philebus, Introduction by Prof. Jowett, 2nd edition, p. 29 et sqq.

only one species of non-hedonistic impulse—that towards the happiness of others—is rational. Physical investigation, by proving that this particular species of non-hedonistic impulse, owing perhaps to its *similarity* to hedonistic action, its quasi-volitional character[1], has an advantage over other species, might present a circumstance[2] determining the intuitivist to pursue exclusively the privileged species.

Still, if the intuitivist insist that, on the one hand, he finds it ultimately rational to seek his own greatest good; and on the other hand, he is determined by categorical axiomatic imperative to pursue the utilitarian end, irrespectively of his personal happiness, and with a degree of self-abandonment which no start from the position of egoism can effect, it seems that Physical Science can give no help[3], unless we entertain a third supposition, swim through a third wave, the greatest of all, and likely[4] *γέλωτι, ὥσπερ κῦμα ἐγγελῶν, καὶ ἀδοξίᾳ κατακλύσειν.*

III. Let it be granted that physiology could ascertain the physical conditions of rightness, the Baconian form of duty! Considering that the cerebral conditions of speech[5], or at least of its absence, and of the intellectual debility attending aphasia, are already to some extent investigated, that the faculty has been connected with a definite locality in the cerebral hemisphere; it may be said to be within the verge of possibility that, as the physical seat of λόγος in its lower sense is ascertained, so also the throne of λόγος in the sense of regulative principle should be ascertainable. Upon the hypothesis of the interdependence of body and mind (it might be argued) all the generalizations of reflection are reduced to the level of empirical laws[6], presumed to be resolvable, though not yet resolved, into laws more general and more certain. It is an incident of

[1] This suggestion is due to Mr. Sully.

[2] On almost any intuitive theory, perception of what is right depends on knowledge of *certain* circumstances. Supra, p. 9.

[3] "Following nature" not being a helpful standard. See Note D.

[4] Plato, Republic, v.

[5] Wundt, Physiolog. Psychol., p. 288. Cf. Maudesley on Aphasia.

[6] See note E.

empirical laws that beyond the narrow limits of their fulfilment, there is often a region of apparent lawlessness, where now one phenomenon appears, and now its contrary, without any assignable reason, owing to some change in the unknown causes, or their collocation. Thus, in the familiar instance of motion round a centre, there was, or might have been, a region of apparent lawlessness, where now was presented the phenomenon of a closed, and now of an infinite curve, while as yet the reason, a variety of initial circumstance, was not assigned. How long would a race, destitute of the necessary mathematical conceptions, and without the means of copious experiment, have groped in ignorance of the principle which unifies these phenomena! Or, if the problem had been to find the direction of a needle attracted by two poles of a magnet, how long, under the same disadvantages, would a general theory of the composition of the forces have been wanting; though the direction might soon have been empirically ascertained in positions where the attraction of one pole might be neglected? Now, if the impulses of self-love and duty, whose opposition has been so well exhibited by Mr. Sidgwick, were dependent on opposite physical forces, with what prospect of success should we seek for the law of their composition by unaided introspection? How much better would introspective observation progress, if we had the slightest clue or hint as to the physical basis; for in the words of the philosopher[1], to whom we are indebted for the best estimate of the evidential value of "consilience," "An amount of knowledge quite insufficient for prediction may be most valuable for guidance."

But, indeed, the preceding examples of the value of deduction, are much too favourable to unaided empirical observation. Suppose rather the primitive observations to have taken place in some dusky subterranean region with sense of eye-sight half-evolved, perceptions *μεταξὺ τοῦ εἰλικρινῶς ὄντος καὶ τοῦ πάντως μὴ ὄντος.*

It is unnecessary to pursue these suppositions further, far beyond the limits of scientific hypothesis. Physical Ethic

[1] Mill, "Logic."

is seen to be a frail edifice resting on a secure basis, the fragility of the edifice rapidly increasing with its height. Frail as it is, it may be well to shew that it is not overthrown by certain objections which it is usual or obvious to urge.

I. It is objected, both vulgarly and by high authorities, that between physics and morals there is a great gulph, so that the former kind of knowledge can never pass into the latter. But surely the whole scope of deductive science, and especially of applied mathematics, is to deduce from laws about one set of phenomena propositions about phenomena quite disparate; given certain intermediate propositions, like bridges with an extremity in each of the separate domains, for example, such and such oscillations of air impinging under certain conditions upon the ear are conjoined with (produce) such and such sound impressions[1]. Thus it is deduced (in certain cases) that, white light diverging from a centre and passing through a small aperture, there will be colours arranged in a definite order on a screen properly placed. Thus, again, the *psychological* laws of association, which Mill places in the basement of his three-storied edifice of the science of human nature, are seen themselves to

[1] The type of the premises is

$$A\,B\ldots \text{ conjoined with } X\,Y\ldots$$
$$A\,B--\text{ conjoined with } \alpha\beta--,\ X\,Y\ldots \text{ with } \xi\upsilon\ldots$$

where the Roman and Greek letters denote the disparate phenomena. That the conclusion should consist exclusively of one set of phenomena (denoted by the Greek letters), is not, I think, a general case. In mathematical physics space naturally appears in the conclusion as well as in the premises. The following is, I think, a valid, though certainly not very valuable, deduction from the mathematical theory of atmospheric vibrations. Simple notes, which afford each a *strong* sense of concord with the same third, afford *some* sense of concord with each other. For their waves are of the form $A_3 \sin \frac{2\pi}{\lambda_3} t$, $A_2 \sin\left(\frac{2\pi}{\lambda_2} t + B_2\right) A_1 \sin\left(\frac{2\pi}{\lambda_1} t + B_1\right)$: where $\lambda_1\ \lambda_2$ have each a *very* simple ratio to λ_3, and therefore *at least* a *rather* simple ratio to each other; and, therefore, afford some sense of concord (including unison) with each other. Here is a conclusion quite independent of space and number. No doubt, like so many deductions, it expresses only a *tendency*, and must be accepted with reservations. One is that, as Wundt says, "der einfache Ton ist nur ein Gegenstand der Abstraction dem aber allerdings gewisse Klänge, in hohem Grade sich nähern."

have their "dark foundations deep" in physical and psychophysical laws; physical, that certain molecular movements are facilitated by repetition; and psychophysical, that the presence of ideas in consciousness is conjoined with the molecular movements of definite portions of nerve-matter: to indicate briefly and obscurely what has been stated with more or less precision by Herbert Spencer and others, by none more elegantly than by Mr. Barratt[1]. So, again, the intensity of sensations is deducible from the intensity of stimulus, according to the Fechnerian law; and in so far as the Fechnerian or a similar law (to be discussed in the sequel) is applicable to pleasures, the conditions favourable to the production of the greatest quantity of pleasure from a given stimulus may be deduced[2]. If sounds and colours, if ideas and sensations and pleasures, are amenable to physical science in the sense explained, why not also duties?

II. It may be objected that this whole way of enquiry, and particularly the first and third suppositions[3], presume an experiential theory of the origin of knowledge—both that general ethical propositions are given by experience personal or ancestral, and that the ethical predicate (right, duty, &c.) is generated out of simpler ideas or feelings, in the course of evolution, by a sort of mental chemistry. But such a theory involves, as Mr. Sidgwick has shewn, no prejudice to the intuitive method of morals (in his sense of the term intuitive, the only one of practical interest). And, as the experiential theory of knowledge has been freed from many difficulties by the qualifications introduced by Herbert Spencer, so in the opinion of many it bids fair to become the catholic doctrine. However that may be, considering the facility with which *à priori* systems have claimed and adopted principles *when discovered by induction*, e.g. the conservation of energy, like wise patriciates incorporating the plebeians who have won their way, there seems no doubt that, if the physical proofs under consideration really were forthcoming, there would be no serious metaphysical difficulty about their employment.

[1] See his original explanation of the law of similarity by that of contiguity. ("Phys. Ethics," Appendix.)

[2] Infra, p. 40, et sqq.

[3] Supra, pp. 11, 16.

III. Much the same remarks may be applied to the objection that the ethical propositions, being categorical imperatives, are not amenable to deduction. It seems sufficient to reply, that they *are* propositions. Whatever your practical predicate (right ought-to-be-done, &c.), it cannot be sustained that a set of physical marks consilient with the judgment of consciousness would not be a great help in the case of confused and obscure consciousness. It might equally be objected, that advice in case of doubtful duties is useless. It is admitted by Mill, who has insisted very strongly[1] on the distinction of scientific and practical propositions, that the latter are amenable to some sort of proof, some sort of evidential consideration[2]; and surely the physico-ethical positions (if confirmed) might well be among such considerations.

IV. The remarks[3] of Mill on the complicity of human nature, refer rather to sociology in all its breadth, than to the narrower enquiry before us; and, as far as they are applicable to the latter, are to be met by Mill's own principles, that a degree of knowledge quite insufficient for prediction may be valuable for guidance, and that much is gained for the observer if he can deduce *à priori* what is possible. He will then know, at least, what to look for.

V. It may be denied that the necessary cerebral investigations are possible. You can't vivisect a moral agent. But, as Wundt says, most of our present (not inconsiderable) knowledge about the physical conditions of intellect and volition is derived from other sources, "Vergleichende Anatomie, Vergleichende Untersuchung der Individuellen Unterschiede des Menschichen Hirnbaus[4]."

VI. It may be said that the knowledge, even if attainable, would be useless. A man can't lower an aretometer into his brain, to see if he is doing right. This is only No. I. over again. Sir Henry Holland and others hold that, on purely physical grounds, a certain amount of cerebral activity is beneficial to the organism. If a man wishes

[1] "Logic," book vi. sub finem. [2] "Utilitarianism," p. 6. See infra, p. 29.
[3] Book iii. c. 24, § 9, and book vi. passim. [4] Wundt, "Physiolog. Psychol.," p. 223.

to avail himself of this opinion, must he open his brain to see if it is working? or would the introspective marks of brain activity suffice?

VII. It may be said that in the fluctuating state of ethical conceptions the investigation of their physical conditions is impossible. When Newton investigated the physical conditions of sound, he had a clear idea of sound. Even when Bacon[1] started on his chimerical search for the form of heat, he had, at least, a distinct sensation of heat. This objection does not come with much grace from the person most likely to make it, the intuitivist, professing to have clear ethical conceptions. It might be observed that it would still be possible, and might be useful, to investigate the laws of atmospheric waves, whether the aery oscillations were sounds for ears to hear, or dumb vibrations for organs less evolved. Or, to take a metaphor which is almost an example, if conscience be compounded in some such way as Mackintosh suggested, out of perceptions of the expedient, then, though there may be much confusion of view as to the precise appearance of the compound, still, on any such view, investigations of the expedient are important.

VIII. There is another class of objections more likely to be felt than to be well expressed by the plain man (to use Butler's term), who, hearing that his notions of beauty and virtue are being resolved into matter and motion, is apt to feel as the converted anthropomorphist: "They have robbed me of my God!" And it must be admitted that morals have too good reason to beware of physics: so inveterate is the tendency of physicists to assert that nerve-movements, &c., *are* feelings in some more intimate sense than that the phenomena are conjoined: so obstinate is the vice of materialism, so wide-spread, and so gross. But since this fallacious tendency of the human intellect entering upon psychophysical science has been abundantly exposed by the ablest writers, (by Mill, for instance, who

[1] Is not Mill a little hard upon Bacon, "Logic," iii. 22, § 4. Are not the physical conditions of light, sound, heat, &c., as ascertained by modern science, very much what was desiderated by Bacon?

treats it as a "fallacy of simple inspection[1]," and by Lange throughout his "Materialismus,") there is hope that this fallacy may be guarded against as well as any other "*à priori* fallacy," which besets the progress of science. Morality might be no more injured by physical science than music by acoustics.

To conclude: It is difficult to assign an objective value even to a scientific hypothesis, much more to one confessedly of the slenderer kind. The same intellectual temper which doubts that the existence of ether is or can be proved[2], may doubt even the interdependence of body and mind[3]. The further suppositions which Physical Ethic requires will appear to many the *reductio ad absurdum* of Physical Ethic: while to some it may seem that a faint light from sources of physical derivation begins to shine upon the dark spot of ethical enquiry; that, though the conclusion, or the inconclusiveness, of the "Methods of Ethics" be accepted now, and for the present generation, the efforts of philosophy are *not* "foredoomed to an inevitable failure;" the "Cosmos of Moral Science" *not* "reduced to a Chaos[4]," though the greatest master of introspective analysis despairs of unification.

SECTION II.

But if the dawn of Physical Ethic is not yet at hand, we ought not in the meanwhile to neglect the domestic light of introspection. One of the most brilliant sources of such light, the "Methods of Ethics," is to be examined in this section. Without presuming to estimate the almost inestimable benefits conferred upon philosophy by this great work, I shall attempt with great diffidence to elucidate some points in it, especially such as seem, either from the characteristic obscurity of reflection, particularly to require, or, in the light of recent researches, most likely to receive, additional elucidation from physical science.

[1] "Logic," v. iii. [2] Cf. Mill, "Logic," iii. c. 14, § 6, p. 20, eighth edition. Ib., chap. 20, § 3. [3] "Logic," vi. 4, § 2. [4] See the melancholy conclusion of the "Methods of Ethics."

THE TERMS, "RIGHT," "OUGHT," "REASON."

I. It is a little unfortunate, but perhaps inevitable, that the terms, "right," "reasonable," "what-ought-to-be-done," &c., should be employed to designate the predicate of the pure egoist's practical proposition. For the pure egoist nothing is to be done but what, in a cool moment, when he has made up his mind, it is pleasure to do; what he then wills to do (what, by the uniformity of nature, others in the same circumstances tend to will). The only origination of action which he (*quâ* egoist) recognizes is volition, in Mr. Bain's sense, including the stage of "conflict of motives." His only impulse (in calm hours[1]) is desire of his own pleasure—pleasure now in objects of sense, now in the pleasures of others, now in (the contemplation of) some state of his own desires[2]. But he is apt to think that some further admission is being forced upon him, when he reads that the "notion of ought is a necessary form of our moral apprehension, just as space is a necessary form of our sense-perceptions." Language like this has so often been used of "right," &c., in the sense to which the egoist denies recognition; the reason originating action combated by Hume; or some notion quite distinct from "the pleasurable," of which Stewart perhaps has given the most palpable account, when he compares it to the relations, equality, identity, &c. The egoist is offended by such language as "the imperative, inhibitive, coercive effect of the moral ideal[3]." But the impartiality of the "Methods of Ethics" is not compromised by this terminology. Judgment on the "Method of Egoism" is summed up with impartial candour, though in the opening forms of the trial some favour might seem to be shewn to the other side. No favour is shewn to either side. For,

[1] Supra, p. 9.

[2] Mackintosh says that Butler should have derived the *supremacy* of conscience from its being a desire of desires. Probably Butler meant more than Mackintosh, and Mackintosh more than the pure egoist. After all, can any one unhesitatingly distinguish in consciousness such differences? Are they not like indistinctly-felt differences of temperature, as to which one is disposed to accept the verdict of the thermometer?

[3] "Methods," book i. c. ix. § 1.

indeed, it may be suggested that in accomplishing the difficult, and never before attempted task of surveying with equal eye the Methods of Egoism and Intuitionism, it was impossible to avoid some compromise of terminology; so that, as something is occasionally added to the predicate of the egoist, so something is occasionally taken from the predicate of the intuitivist. At least, in the circularity attributed to so many of the intuitivists[1] this may seem to have taken place. To start in search for the "reasonable" in the thinner Sidgwickian sense, and to find that it is the "right" in the fuller sense of Stewart or Price, to say nothing of the more impalpable Germans, is not necessarily a circular proceeding. The answer *might have been* the "pleasurable." With regard to Butler in particular, it may be shewn on somewhat different grounds that the argument of the Sermons is not quite so viciously circular as the "Methods of Ethics" represent[2].

Freewill and Necessity.

II. The interdependence of body and mind, as postulated by Physical Ethic[3], may suffice to clear up another of the ἀπορίαι of our English Aristotle:—

"This almost overwhelming cumulative proof seems, however, more than balanced by a single argument on the other side (that of freewill), the immediate affirmation of consciousness in the moment of deliberate volition. It is impossible for me to think, at such a moment, that my volition is completely determined by my formed character, and the motives acting upon it[4]," &c.

But if both motive and action are not cause and effect, but *co-effects* of the same physical causes, then we should no more expect action to have conscious motive as an invariable antecedent or concomitant, than (to adapt a metaphor of Huxley's, which rather illustrates than ennobles) we should expect the motion of the steam-locomotive to be always accompanied with the steam-whistle. If the cause of action is not in consciousness, then action may obey the law of causation, though consciousness discerns

[1] "Methods," book iii. chap. xiii. § 2. [2] Ibid., iii. xiii. § 2. See Note D.
[3] Supra, p. 10. [4] "Methods," book i. ch. v. § 3.

no cause; the doctrine of necessity is not damaged, though even a Sidgwick may have swept the universe of consciousness with the microscope of introspection, and found not everywhere a cause.

The observations quoted from the "Methods of Ethics" are doubtless pertinent to, and probably decisive against, that other form of necessarianism in which the invariable antecedents are, as Mill says, "spiritual[1]." It is remarkable that Professor Clifford, in his smart remarks on Mr. Sidgwick's doctrine in an article on Right and Wrong in the "Fortnightly Review," should have ignored this form of necessarianism, Mill's form.

Pleasure and Desire.

III. It has already been suggested that non-hedonistic preference (if such there be, exclusive of habit proper) is ancestral habit[2]; which it is submitted nobody is either competent by the light of reflection, or concerned for the sake of virtue, to deny.

Differences of Kind in Pleasure.

IV. He who accepts the form of utilitarianism[3] stated in the "Methods of Ethics," must be prepared to accept the dictum of Bentham, "Quantity of pleasure, being equal, push-pin is as good as poetry;" the sarcasm of J. Grote, that there must be a general scale of pleasure, in which so many marks will be given to drunkenness, so many to the love of arts. *Οὐ φοβητέον τὰ τῶν χαριέντων σκώμματα.* When Hume, in treating of the domestic relations[4], balances so much chastity against so much of the pleasures of social commerce, our utilitarian may deplore with Mackintosh that the measurement of pleasures should have been so incompletely performed by Hume. But he must not with Lecky protest against the *μετρητικὴ* itself[5].

[1] Exam. of Sir W. Ham., "Logic," book vi. chap. iii. [2] Supra, p. 11.

[3] The following remarks are applicable, *mutatis mutandis*, to hedonism generally. "Methods," book i. ch. vii. § 2.

[4] Dialogue at the end of Inquiry concerning the Principles of Morals. Essay on Polygamy and Divorces.

[5] To avoid misconstruction, it may be well to say, that nothing in this paragraph is calculated to disparage the domestic virtues; in favour of which the

That other species of utilitarianism which recognizes kinds of pleasure is, as Mr. Sidgwick has remarked, not to be distinguished from intuitivism. Let any reform be proposed, any question of casuistry stated, qualitative utilitarianism can always take up the position, that certain advantages (endeared by custom) are incommensurable with any quantity whatsoever of proposed gain. What, then, do such utilitarians more than the intuitivists?

Between the paradox of the former doctrine and the orthodoxy of the latter may there not be a *via media?* The paradox, it may be observed, often results because, in the comparison made, a sufficient volume, so to speak, of the pleasure of inferior specific density is not taken. No multiple of the pleasure of eating tarts, says Lecky, can be equated to the pleasure of doing a generous action. Perhaps not, in the volumes usually compared; the amount of tarts which an individual eats, the action which an individual does. But query whether a tax imposed on the poor man's sugar, to afford a prince the means of performing an act of graceful generosity, would be (abstraction being made of the injustice) felicific? Extending this view, one might suppose that the so-called lower pleasures are related to the higher, somewhat as differentials to an integral, incommensurable indeed, yet capable of being equated after infinite summation. The permanent increase of material comforts and pleasures over an indefinite area of society, and through countless generations, may be set off against a definite and limited dereliction of moral beauty. It may be objected that, in taking account of the alternative consequences to society and posterity, we must integrate also the higher pleasures which the supposed dereliction tends to destroy: so that

utilitarian balance had been already turned by Hume (Essay on Polygamy and Divorces); and now the weighty considerations adduced by Mr. Sidgwick are to be added ("Methods," book iv. chap. 3, § 6). Add also the serene philosophy of Mr. Jowett (Introduction to Plato's Republic, second edition, p. 162); and Mr. Bain's profound psychological proof that "Ideal Emotion," the deeper affection, best flourishes when it clings round one loved object. ("Emotions and Will," third edition, chap. v. § 11).

the two integrals will still be incommensurable. Nevertheless, instances might be adduced of customary practices, rooted and intertwined in the higher feelings, which might, however, be modified with no permanent shock to morality, and with a permanent accession of material advantages. Such a practice may be the abstaining from meats on religious grounds. Such, again, was the ancient Roman prejudice (quoted by Lecky) against women drinking wine. And it might not be difficult to add instances from the manners of the Romans, or of any other people, who mistake prejudices for virtues.

It may be said that the differentials in question are no more (or rather no less) than very small quantities, which may practically be neglected in comparison with other quantities; so that theoretically Benthamism is untouched. There is room, perhaps, for two shades of opinion; all that is here contended, is that the different views of the "plain man" and the theoretical utilitarian, in reference to the commensurability of pleasures, may be accounted for by the different extent over which they perform the summation of inferior pleasures.

The Method of Egoism.

V. Some exception may be taken to Mr. Sidgwick's low estimate of the value of authority as a criterion of one's own greatest pleasure. In details, no doubt, it is shewn how futile it is to take care of the pence, so to speak, of one's own happiness. But with regard to the adoption of general lines of conduct, the authority of the χαρίεντες may afford more than commonplace advice to common men[1]. It may be maintained, on the principles of Hume's Essay on the Standard of Taste, that reliable authorities are practically attainable. They are such as have compared most objects (of beauty or pleasure), have the greatest fineness of perception, intellectual discernment[2],

[1] "Methods," book ii. chapter iv.

[2] Hume mentions two reasons why intellectual discernment is a criterion of a good authority on the standard of taste. One reason turns upon the perception of means to end, which hardly applies to the subject here under consideration, the choice of an end. As to the other reason, see Note C.

and so forth. They are easily denoted, though with difficulty defined. Thus a standard, which the greatest number, or rather those who have paid most attention to the subject, tend to accept, is attained. It is sufficient here merely to refer to this well-known and important Essay; in the light of which, it is submitted, half the difficulties[1] about "*objective*" beauty (and indeed objective truth) disappear. The great author designed the application of his principles to the pleasures of virtue[2] also, as to which (as above intimated) he expected unanimity rather in the main features than in details[3].

Again, exception may be taken to a sort of *à priori* assumption, that egoism as the ultimate reason cannot lead to utilitarianism as the method, a deduction from the very meaning of the terms, that "a man cannot both wish to secure his own happiness and be willing to lose it[4]." For it is not *inconceivable* that one's greatest pleasure—in those calm moods which give the character to one's life—should invariably consist in the contemplated pleasures of others: a genuine hedonistic desire which, though a hedonistic desire, though "self-love serves to wake" it, yet

"takes every creature in and every kind."

It is possible to conceive a soul moved only by the attraction of hedonism, so moved that the integrated attractive force of the pleasures of all other souls, an attraction *not* varying with the social distance, should constitute an acceleration in the direction of utilitarianism, compared with which any other motive, any other hedonistic force whatsoever, might be neglected, or rather could only act in directions not interfering with the great resultant.—There is no more difficulty in conceiving such a standard of emotion, than the following standard of opinion; which is so far from being inconceivable that it actually has

[1] Cf. the objections of Mr. Bain and others, as to the difficulty of finding the σπουδαῖος. "Mental and Moral Science," *s.v.* Aristotle.

[2] Essay on the Standard of Taste, &c.

[3] Ibid., Dialogue at the end of Inquiry, sub finem.

[4] "Methods," book ii. chap. 5.

existed. Suppose a person to have made up his mind to believe what the Church[1] believes. It is in vain to object, "But what if your opinion should differ from that of the Church;" *ex hypothesi*, the difference will disappear as soon as discerned.

In fact, Mill's proof of utilitarianism may seem to rest upon the two considerations just noticed; the tendency towards a uniform standard (as explained by Hume) of pleasure or desire; and the (alleged) fact that this standard desire turns out to be the desire of the general happiness, above shewn to be conceivable. The argument is "inductive." One makes up one's mind about the standard by *observing* what pleasures are greatest, even by *experimenting*[2]. The latter view might be taken of such remarks as, "A person in whom the social feeling is at all developed, *cannot bring himself to think* of his fellow-creatures as struggling rivals with him for the means of happiness, whom he must desire to see defeated[3]." Such, perhaps, is the cogency of much that is said about the future of mankind. By *imagining* a position where the inequalities of legal privilege between classes or individuals are levelled, and other changes to which society may be tending (pp. 48—50), we may discern that the utilitarian motive is general and deeply rooted; the cases in which it does not act temporary and exceptional. The discourse of the philosopher, putting it to one whether such and such is one's pleasure, assists the inductive process, as the demonstration of the experienced anatomist assists the unskilled observer. Of such a kind, perhaps, are those "considerations capable of determining the intellect," but not "in the way of intuition[4]." Also, the inductive process may consist of that copious, internal, and even unconscious species, by which the generalizations of mathematics are

[1] Or any other consensus of ultimate authority.

[2] A good instance of what may be called a moral experiment is Hume's supposing both a Saturnian age and a (Hobbist) "state of nature," and then examining what would be our feelings under these supposed circumstances. Inquiry, Justice.

[3] "Utilitarianism," p. 50.

[4] Ibid., p. 6.

obtained. In that case, as critics have observed, the evidence of ethical propositions, though called inductive, would be of the highest nature. The proof, then, of utilitarianism, from Mill's point of view, consists essentially in making up one's mind by comparison of one's pleasures. To this proof it might be objected, that moral action is determined, not by way of hedonistic desire, but is originated by reason, prescribed by *à priori* categorical imperatives, &c. Such an objection, then, is anticipated by the doctrine which Mill enforces in his "Proof," that "will is entirely produced by desire." As for "sanctions," they are rather adventitious pleasures connected with utilitarian action, than that deep immanent pleasure which "Proof" exhibits. They are (as Hume would say) rather the dower, than the charms, of utilitarian virtue. (But the distinction is hardly kept in sight by Mill.) It is submitted that this view of Mill's proof, though requiring certainly rather a violent effort of μαιευτικὴ, equally with the interpretation given in the "Methods of Ethics[1]," is yet more consonant, if not with the language of Mill's Utilitarianism, at any rate with his position in philosophy as the disciple of Bentham, and the heir of Hume.

Equipped with the principles of Hume and Mill, let the egoist choose his course of life, arranging in a quiet moment the points upon the line of action, so that the trains of volition may sweep by without delay and accident. Then, perhaps, it may appear that the preceding considerations are sufficient to establish Hume's principle of utility, non-quantitative, and apparently not requiring the last self-sacrifice. But they do not seem adequate to sustain the more precise and severe utilitarianism of Mill. Though, at the same time, there are not wanting appearances of approximation to the sublime desire of universal happiness, above described as not inconceivable; however ridiculous it may appear to the false friend and the dire foe of hedonism, to the voluptuary and the Hegelian. And quite appropriate in an egoist of the purest type[2] are the sentiments and glowing words of our author[3].

[1] "Methods," iii. c. 13, § 5. [2] Supra, p. 9. [3] "Phys. Ethics," p. 160.

"To see himself in every part of nature, and throb with the universal pulse; to extend his sympathies to the utmost bounds of his knowledge, and be content with his allotted share of the common good; and finally, losing himself in his own infinity, to absorb thought in action and action in thought, subject in object and object in subject; and thus at last, by a true apotheosis, to die into Deity: this is the sublime conception of man's nature and destiny."

Perhaps we may advance the egoistic approach to utilitarianism, if we supplement Hume's standard of taste with Mr. Sully's æsthetical criterion[1], that the highest taste is that which, as he would say, is in the line of evolution. For that an approximation to the utilitarian desire is in the line of evolution is sufficiently probable, the sympathies of men being "widened with the process of the suns." But, in order to have a hedonistic attraction to move in that line, we must dispute Mr. Sully's suggestion, that the boor has as much pleasure as the heir of all the ages. And perhaps it will suffice to refer to Mr. Sully's own copious and most inspiriting enumeration of the sources of happiness specially open to the cultivated will and developed intelligence[2]. Indeed, he has shewn that if meliorism, the creed of progress, fails, utilitarianism also fails. To be sure, that is not much of an argument to the egoist; but it may be an argument, if he has already adopted utilitarianism in some form, upon the strength of prior considerations (for instance, the preceding). Accordingly, the large mass of modern sentiment, which is leavened with a utilitarianism more or less impure (more or less having egoism as its ultimate reason), is coloured with the hope that the progress of civilization is conducive to the happiness of mankind[3]. And, though the highest authority on evolution has expressed himself with caution on the point[4], it does not seem an unwarranted assumption[5],

[1] Sensation and Intuition, Essay XIII., developing Herbert Spencer, "Psychol.," § 539. [2] Ibid., "Pessimism."

[3] Supposing that there is implicit in popular sentiment a proposition not identical. As to the connotation of evolution, see Herbert Spencer, "Psychol."

[4] Herbert Spencer, "Psychology," §§ 215, 539. [5] Infra, viii. 3.

that the pleasures of a higher evolution are the more pleasurable. More pleasurable, it must however be added, rather ἁπλῶς than ἡμῖν; given the conditions to which we may be tending, but have not yet arrived. The extended sympathies, as pictured by Mill among the Sanctions of Utilitarianism, would be more pleasurable, given the millennial social conditions which Mill foresees; but otherwise not, as follows from the masterly reasonings of Mr. Sidgwick[1]. Nevertheless, although no jot or tittle of those reasonings can be set aside, there still may remain a margin of presumption, that the enlightened egoist will "find his account" in *tending* at least, if not moving, in the line of evolution. The flower, too rashly opening to the early spring, is blighted. The lingering bud, cut off unopened by the hand of fate, foregoes the bloom it well might have enjoyed. There is a happy seasonable germination; the bursting blossom of utilitarian love, with the protecting sheath of worldly wisdom.

If there is any solidity in the reflections just made, they may serve to blunt the edge of many of Mr. Sidgwick's observations, which turn upon the *rarity* of utilitarian virtue.

"It is doubtful whether *many* men would affirm even this, (that there are "one or two human beings so dear to him, that the remainder of a life saved by sacrificing their happiness to his own, would be worthless to him from an egoistic point of view")[2].

"There are *very few* persons, however strongly and widely sympathetic, who are so constituted[3]," &c.

"It must surely be admitted, that there are *comparatively few* men in whom morality has reached anything like this pitch of development[4]."

"In order to constitute such conduct reasonable, we have to assume that in *all* cases where such a duty could exist, or at least be recognized, the moral pain that would follow on evasion would be so great, as to render the whole remainder of life hedonistically worthless[5]."

[1] Book ii. ch. 5; iv. ch. 6. [2] Book iv. ch. 6, § 3. [3] Ibid.
[4] Book ii. ch. 5, § 4. [5] Ibid.

"Can we say that *all, or even most,* men are so constituted, that the satisfactions of a good conscience are certain to repay them for such sacrifices[1]."

But, even upon Hume's principles, the *number* of authorities on a question of pleasure-value is by no means their most essential recommendation. Much more upon the principle developed by Mr. Sully. The crest of the advancing wave of evolution is not considerable ὄγκῳ. The desire under consideration (that of universal happiness), though not general, may be nascent among the χαρίεντες and σπουδαῖοι: who, on the principles of the authors quoted, are discoverable, though not definable. The egoist, then, may have the power and the motive to cultivate a desire[2] for the general good, not, indeed, to the pitch of pure utilitarianism, but to a degree asymptotically approaching it, in the progress of evolution, in the course of generations.

Philosophical Intuitionism.

VI. After the egoistic may be considered the intuitivistic approach to utilitarianism.

Mr. Sidgwick has not affected logical precision as to the method of the process from common sense general rules, to philosophical intuitionism. If the former are attained by induction from particular propositions[3], then the latter presumably is a case of what is called explanation in inductive logic; and the true, the just, the kind, would be subsumed in the felicific, as terrestrial in celestial gravitation. But, if the general propositions of common sense are *à priori*, would their unification be a process of the nature contemplated by Sir William Hamilton, when he says, that the function of philosophy, in reference to the dicta of consciousness, is to reduce them to the smallest number? Leibnitz also

[1] Book ii. c. 5, § 4. The italics in these quotations are not Mr. Sidgwick's.

[2] Both the æsthetic criteria described in the text may be employed by the egoist, to estimate the pleasure-value both of hedonistic and non-hedonistic desire. In fact, Hume employed his standard to estimate "sentiment," and his sentiment was partly non-hedonistic. See Note C.

[3] "Methods," i. ch. 8, § 2.

speaks with satisfaction of reducing two geometrical axioms to one. Or is the process of the nature of the so-called perfect induction? Or, if it be imperfect induction, would à priorists allow more weight to this kind of induction from intuitive generals, than they do to ordinary induction, which starts from intuitive particulars [1]?

By the construction of philosophical intuitionism, the coping-stone appears to have been imposed on the time-honoured edifice of intuitive morals, — a secure height, neither shaken by the winds of conflicting custom, nor vanishing in the clouds of metaphysical ethics. Here, all who, with Plato and the Stoics, with Kant and the followers of Butler, think that right is to be done irrespective of consequences, at last may discern *what* it is right to do. What was divined by Cumberland, what was foreshadowed by Hutcheson, what was desiderated by Stuart Mill [2], the noblest of the hostile school, now, at length, appears consummated — intuitive utilitarianism. The method of Kant, the creed of Mill; practical reason imperatively prescribing the utilitarian end, "desirable conscious life for the innumerable multitudes of living beings present and to come [3]." * * * *

The reconciliation of self-sacrificing duty and self-love was sought by Butler in religion: and perhaps it will be sought by Mr. Sidgwick [4]. And possibly, as the opposition might have been predicted [5], so the composition may be calculated by Physical Ethic. In the meanwhile, Mr. Sidgwick has shewn that, if intuitive utilitarianism gives way, we must fall back upon egoism, (common sense affording no resting-point): and Hume, and Mill, and Sully [6], may shew that egoism is capable of self-devotion, above the pitch of the Butlerian self-love, and above those "pleasures of virtue and benevolence" allowed by the "Methods of Ethics" to rational egoism.

[1] Mill, I think, confines the function of intuition to particular propositions.
[2] "Utilitarianism," pp. 4, 44. [3] "Methods," iii. chap. xiv. § 2.
[4] Ibid., sub finem. [5] Supra, p. 13. [6] Supra, V.

Meaning of Utilitarianism.

VII. Taking for granted as the standard of morals, utilitarianism, (provisionally, at least, and except in cases of extreme conflict with egoism,) let us proceed to consider the particular form of utilitarianism adopted by Mr. Sidgwick. According to him, the utilitarian end[1] is the greatest quantity of happiness of sentients, exclusive of number and distribution—an end to which number and distribution are but means. Not differently, though not quite so distinctly, is the utilitarian end defined by Fechner in his charming treatise "Ueber das höchste Gut." The problem of distribution Fechner illustrates by the problem to divide a given number into a given number of parts, so that the product of the parts should be a maximum. It is curious that what is only an illustration in Fechner's earlier work, becomes a sort of proof in his later Psychophysic[2].

The doctrine of Fechner and Sidgwick may be termed exact utilitarianism, as distinguished from Hume's non-quantitative principle of utility, and the not very explicit greatest-happiness principle of Bentham and his followers, including J. S. Mill. The purport of this and the following paragraph is, to confirm exact utilitarianism; first, by shewing it to be implicit in the greatest-happiness principle; and secondly, by deriving from it (through a mathematical channel) conclusions consistent either with common sense or common utilitarianism.

[1] "Methods," book iv. chap. 1, § 2. "The point up to which, on utilitarian principles, population ought to be allowed to increase, is that . . . at which the product formed by multiplying the number of persons living into the amount of average happiness reaches its maximum," &c. The remarks which follow as to the utility, notwithstanding the apparent absurdity, of abstract precision are commended to my reader,—"That our practical utilitarian reasonings must be rough is no reason for not making them as accurate as the case admits; and we shall be more likely to succeed in this, if we keep before our minds as distinctly as possible the strict type of the calculation that we should have to make, if all the relevant considerations could be estimated with mathematical precision." [2] Infra, p. 40.

First, then, what can be the import of seeking "the greatest happiness of the greatest number?" Let us approach the question by considering generally what is the meaning of the problem, to find the greatest quantity of one thing (U) *of*, or in relation to, the greatest quantity of another thing (V).

(1.) U and V being functions of the same variables, the problem may mean to find the variables, so that U should be the greatest possible, and then to investigate whether the same solution happens to make V the greatest possible. Thus, if U and V were functions of a single variable, it might happen that the same value of the variable satisfied both the equations $\frac{dU}{dx} = o$ and $\frac{dV}{dx} = o$, and corresponded to the greatest possible value of U and V. And the calculus of variations might present a similar case (two differential equations having a common factor, and other conditions favouring). A guide-book (for the neighbourhood of Scarborough) says that by a certain route the traveller may obtain the *best* view of a certain tract, with the *least* trouble (*most* ease). I suppose it means; Of all routes which afford a view at all, this is the easiest; and the view afforded is the best of all possible views.

(2.) W being a function of U and V, the problem may mean to find U and V so that W should be the greatest possible; but is most likely to mean some species of this genus.

When one of the variables is regarded as dependent on the other, the problem may become: first, to find the *form* of U, then the *value* of V, that W should be the greatest possible. This is[1] what Strauch calls, "ein Maximum-Werth eines Maximum-Standes."

The case of W being such a function of U and V that W increases with every increase of either U or V is especially appropriate. In this sense we look for the workman

[1] In so far at least as maximum, in its technical mathematical sense, admits of being identified with "greatest possible quantity," a distinction which the reader is requested to bear in mind.

with most strength and most skill, when we look for the *most efficient* workman.

A species of this species is the case of W being the product of U and V. In this sense, we might describe the most efficient machine as that which lifted the greatest weight to the greatest height.

(3.) U being a function of certain variables, and V of the same and certain other variables, the problem may mean; first, to find the first set of variables, such that U should be the greatest possible; and then to determine the *remaining* variables, such that V should be the greatest possible. For example: first, to construct in a given plane a curve of given length, such that the enclosed area should be the greatest possible; it will be a circle *anywhere* in the plane: secondly, to find *the position* of the curve so found, of the circle, such that the column standing upon the circle, and intercepted between the given plane $(x\,y)$ and the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{b^2}{c^2}=1$, should be the greatest possible. The centre of the circle is found to be the centre of the ellipsoid. It is hardly necessary to point out that, if the problem had been simply to find the form and position of the curve (of given length), such that the intercepted column should be the greatest possible, the answer would have been different. The required curve would have been an ellipse, its centre being the centre of the ellipsoid. To this heading may be referred certain indeterminate cases in the calculus of variations (e.g. where the leading differential equation does not involve the dependent variable). A maximum having been secured by the solution of such a problem, the indeterminate constants (or functions) may be determined so as to secure a second maximum (the maximum of some other expression involving the variable). A moral example of this third interpretation might be the case of him who holds—

> "Quærenda pecunia *primum*
> Virtus *post* nummos."

(4.) U and V being functions of the same variable, the problem may mean to determine *some* of the variables, such that U should (tend to) be the greatest possible, and the others similarly for V. Here a few special cases :—

U and V being functions of X and Y, equate $\frac{dU}{dx}$ and $\frac{dV}{dy}$ to Zero.

If $U = \int \phi\,(xy\,\frac{dy}{dx}\ldots)\,dx$, and $V = \int \psi\,(xy\ldots..)\,dx$, determine the *form* of Y so that U should be a maximum ; and the *limits* such that V should be a maximum.

U and V being as before, only with fixed limits, proceed as in the case of an ordinary *relative* maximum to find the form of y so that $\int_{xo}^{x1} (\phi + \lambda\,\psi)\,dx$ should be a max. ; and then determine λ not that $\int_{xo}^{x1} \psi\,dx$ should be a given, but that it should be a max. (differentiate V, as to λ, and observe the critical values). For example ; to find the curve of given length measured from two points, with equal ordinates, through which the curve passes, and such that the area intercepted between the abscissa, the ordinates, and the curve is a maximum. The required curve is a circle ; and the *greatest* length (to be) given which lends itself to the solution, is that which corresponds to a circle tangential to the ordinates. (If the given length exceed this, the solution has to be eked out by drawing a circle touching the ordinates above the points, and considering the ordinates as part of the required curve[1].)

It might be possible to multiply instances from mathematics : but it is evident that we have already exceeded our province,

"That which before us lies in common life."

Rejecting, then, the last case, let us consider which of the three preceding is admissible. The first interpretation is not admissible ; for utilitarians declare that the conditions productive of the "greatest number" are not productive

[1] Todhunter, "History of the Calculus of Variations."

of the "greatest happiness." The second interpretation is admissible in the special sense, that the *greatest product* of number and average happiness, that is the greatest quantity of happiness, should be sought: which is exact utilitarianism. The third interpretation is admissible in the sense, that the greatest average of happiness should *primarily* be sought; *secondarily* the greatest number: a view which might have been attributed to J. S. Mill, but for his not having observed that *ceteris paribus* "the more the merrier[1]." In fact, his statement of utilitarianism is not quite explicit: but it lends itself to the second better than to the third interpretation. For, as it is probable that neither Mill nor any one else would approve of any increase, however small, of the average happiness, attended with any decrease, however large, of number: so it may be shewn perhaps (in the following paragraph) that exact utilitarianism points to that very ideal approved by Mill, a *limited cultivated* society.

Upon the whole, it appears impossible to assign any intelligible or tenable meaning to the formula, "greatest happiness of the greatest number," but that of exact utilitarianism. Thus the utilitarian problem may be illustrated by the following, in which positive motion is put for pleasure, and the material for the moral world. Mass, form of the functions which enter into the equations of motion, &c., being variable, to find, subject to certain given conditions, the mass, form of the functions, and other variables, such that the algebraic sum (integral) of the *quantity of motion in the positive direction* should be the greatest possible. Bentham says "greatest quantity of motion of the greatest mass!" Mill says much the same, adding perhaps to the problem the condition, that the quantity of positive motion should be *the same for all elements*[2] resembling each other in certain respects. Common sense speaks even less precisely about motion. But they all seem to mean what Mr. Sidgwick says, "greatest quantity of positive motion."

[1] Cf. J. Grote on Mill; and "Methods," iv. c. 1, § 2.

[2] "Utilitarianism," 92, 93. Infra.

Proof of Exact Utilitarianism.

VIII. Fechner's law, as Fechner himself points out, supplies a sort of proof that if a given amount of "stimulus" (Reiz), corresponding to a given amount of material means, "fortune physique," is to be distributed among a given set of sentients, the distribution favourable to the production of the greatest quantity of happiness is equality. For if, agreeably to the Fechnerian law, the pleasure of each sentient element is represented by $k(\log \gamma - \log \beta)$, where γ is the stimulus, and k and β are constants, then the sum of the elements is a maximum when they are equal. This sort of proof postulates that Fechner's law is applicable to the intensity of pleasures as well as of sensations; that the co-efficient k does not vary for different elements (the variation of β does not affect the proof); it makes abstraction of the subtlety of our higher nature, of the relations between sensations, of emotions. Now, as Fechner's law has been called in question by many high authorities, and modifications of it have been proposed by Helmholtz, and with great probability by Professor Delbœuf; as there are peculiar difficulties connected with its application to the measurement of pleasures; as the co-efficient k is known to vary; and as Fechner can hardly be said to have reduced the higher complications to mathematical formulæ; it becomes important to consider what sort of conclusions may be deduced from exact[1] utilitarianism, without the above assumptions, and in the more concrete cases.

The sort of proof initiated by Fechner is not peculiar to the Fechnerian law, but may be extended to an indefinite number of similar formulæ, involving, instead of the logarithm, some function which shares with the logarithm the following properties; that the first differential is positive, the second differential negative for all values of the variable, or at least all with which we are concerned. These properties are enjoyed by most of the formulæ[2] which it

[1] Supra, VII.

[2] The not very plausible formula of Paul Langer, $k \log \frac{c\gamma^2 + b}{b}$, enjoys the

has been proposed to substitute for Fechner's law, e.g. that of Helmholtz, $\frac{a}{G-\gamma_0}\log.\left[\frac{\gamma_0+\gamma}{G+\gamma}\right]+C$; and that of Delbœuf, $k\log.\frac{c+\gamma}{c}-k\log.\frac{m}{m-\gamma}$. The properties of the functions express the circumstances that for every increase of stimulus there tends to be some increase of pleasure (sensation), but that, as the stimulus increases, the sensation increases less rapidly (its rate of increase decreases). These circumstances appear to have been assumed by all, including Laplace, who have touched upon the relation of "fortune physique" to "fortune morale;" and they are sufficiently evidenced by every-day experience. It is obvious indeed to remark, that the increase of stimulus beyond a certain point destroys pleasure (and ultimately sensation); that in the curve of pleasure there is a "Wendepunkt," as exhibited by Wundt (and compare Delbœuf's theory of fatigue). But no error will be produced in the following reasonings, by considering the stimulus not to exceed that corresponding to the "Wendepunkt." For where it is concluded that a sentient should have a greater share of stimulus, he is to be considered not as applying the whole stimulus at once to the organ of sensation, but at different times, perhaps to different organs, in appropriate subdivisions. He is not to be considered as throwing all his fuel at once on one furnace, but as lighting up the same furnace, or others, at different times. The first condition then, under proper reservations, may be assumed. As to the second condition, and the second differential of our function, I know of only one consideration which need give us pause, the form of the pleasure-curve as delineated by Wundt[1]; which, after all, may be only a diagrammatical accident. This curve from the "Schwelle" up to the "Wendepunkt," is not continuously concave to the abscissa, as our condition demands; the lower part is convex, the upper con-

first property constantly, and the second property for the higher values of stimulus. γ in the above formulæ is the stimulus; the other signs are constants.

[1] "Physiolog. Psychol.," p. 432.

cave. There is however, I think, reason for supposing that the *upper* part of the curve is alone that capable of being employed in cases of maximum pleasure, and therefore alone concerning us here[1].

Let us then, following in the drift of recent research, assume a quasi-Fechnerian law, $\pi = k\,|f(y) - f(\beta)|$. Here $\pi\,dt$ represents, or is proportional to, the pleasure of a sentient element during an element of time. In summing up the quantity of pleasure extending over a given time (with all due deference to Mr. Green's difficulties about this sort of integration[2]), it is of course generally necessary to integrate with regard to the time. But, as for the present purposes it may be safely left to the reader to take account of the variations of π varying with the time, it will be sufficiently accurate to speak of the pleasure of a sentient element, as proportional to $k\,|f(y) - f(\beta)|$. f is a function enjoying the two properties above postulated; β and k are co-efficients. β denotes the "threshold," the lowest value of stimulus for which there is sense of pleasure[3] at all. Since, the smaller β is, the smaller the values of stimulus for which the sentient has any pleasure at all, or any given intensity of pleasure, the reciprocal of β may denote the "sensibility." There is, indeed, a peculiar propriety in this notation with reference to the particular law of Fechner; but its employment generally seems also not inappropriate, and is countenanced, I think, by the authority of the ablest speculators in this region, Wundt, Delbœuf, and others; who, when approaching the subject from a general point of view, and before they have posited the Fechnerian law in particular, employ the term "sensibility" in connexion with the "threshold." The import of k has not, so far as I know, been quite determined. Let it here be provisionally termed, "capacity for pleasure." The two co-efficients may be considered, and are known to vary with different elements. Some interesting cases presented by their va-

[1] Infra, Condition 2, p. 62.

[2] Note in "Mind," No. VI., Introduction to Hume. See Note F.

[3] The threshold of sensation and pleasure are, according to Wundt, identical.

riation may be arranged under two headings, which might be ambitiously termed, the rudiments of two main problems of the Calculus of Hedonics.

PROBLEM I. Given a certain quantity of stimulus to be distributed among a given set of sentients (with the condition that every element is to have *some* stimulus), to find the law of distribution productive of the greatest quantity of pleasure.

CASE I. Where all the elements are equal as touching sensibility and capacity for pleasure.

The pleasure of the whole may be represented by

$$k\{f(y_1)-f(\beta)\}+k\{f(y_2)-f(\beta)\}+k\{f(y_3)-f(\beta)\}+ \text{ \&c.}$$

where y_1, y_2, y_3, &c., are variables, depending only on the condition that $y_1 + y_2 + y_3 +$ &c. = given. In order, then, that the whole may be a maximum,

$$k\{f(y_1)+f(y_2)+ \text{ \&c.}\}-c(y_1+y_2+ \text{ \&c.})$$

must be a maximum: the solution of which problem (since f'' is by hypothesis negative) is given by the equations

$$kf^1(y_1)=c,\ kf^1(y_2)=c,\ kf^1(y_3)=c.$$

Therefore y_1, y_2, &c., are all equal: the law of distribution is equality.

CASE II. Where the sensibility only varies. By a parity of reasoning, the same conclusion is deduced.

CASE III. Where the capacity-for-pleasure only varies. Here $k_1 f(y_1) + k_2 f(y_2) +$ &c. $- c(y_1 + y_2 +$ &c.) must be a max. The solution of this problem (since f'' is by hypothesis negative) is given by the equations $k_1 f^1(y_1)=c$, $k_2 f^1(y_2) = c$, &c.; or briefly, the equation $kf^1(y) = c$. Since now $kf^1(y)$ is constant, and $f^1(y)$ decreases ($f''(y)$ being negative) as y increases, it follows that the larger values of y correspond to the larger values of k. Unto him that hath greater capacity for pleasure shall be added more of the means of pleasure.

CASE IV. Where both sensibility and capacity vary. By a parity of reasoning, the same conclusion, as in the preceding case, is deduced [1].

[1] Of course, it is taken for granted in all the cases that the data are such as to allow each element a share of stimulus above the "threshold."

In the preceding deductions, the elements have been treated as finite; but the conclusions could equally have been deduced, if the elements had been treated as infinitesimal with regard to the whole. The latter conception appears the more appropriate; since, strictly speaking, each pleasure element consists of the indefinitely small pleasure afforded during a given[1] time, by each indefinitely small element of the whole sensory tract to which the stimulus is applied. (The whole sensory tract is generally considered as made up of tracts belonging to different sentient individuals,—e.g. different animals.) The latter conception (that of infinitesimals) will therefore generally be adopted in the sequel, when it has once for all been observed that the conclusions are equally deducible on either supposition.

PROBLEM II. Given a certain quantity of stimulus to be distributed over some part of a given sensory tract (without the condition that each element of the given tract is to have some stimulus), to find the part and the law of distribution over it productive of the greatest quantity of pleasure.

If at first, for the sake of simplicity, we suppose the co-efficients β and k to be functions of a single variable (for example, the given sensory tract to consist of a rectangle, in which the sensibility and capacity for pleasure vary *only with the length*), the pleasure of the whole may be represented by $\int_{x_0}^{x_1} k\{f(y)-f(\beta)\}\,dx$, where y is the required function, expressing the law of distribution over the required region; x_1 and x_0 are the required limits; and where $\int_{x_0}^{x_1} y\,dx = \text{given}$, say D. In order, then, that the whole may be a max., $\int_{x_0}^{x_1} k\{f(y)-f(\beta)\}-cy\,dx$ must be a max. (where c is a constant, to be determined, after y has been found, by the equation $\int_{x_0}^{x_1} y\,dx = D$).

[1] Abstraction, as aforesaid, being made of variations due to time; e.g. diminution in the impression, owing to the repeated application of stimulus to the same sentient element; a diminution which may, with sufficient accuracy for the present purposes, be supposed the same for all sentient individuals.

CASE I. Where the sensibility and capacity are constant throughout the given tract.

The first term of the complete variation of $\int_{x_0}^{x_1} k\{f(y) - f(\beta)\} - cy\,dx$ is $\int_{x_0}^{x_1} (kf^1(y) - c)\delta y\,dx^1 + dx_1\, x\{k\,|f(y) - f(\beta)| - cy\}_1 - dx_0\, x\{k|f(y) - f(\beta)| - cy\}_0$; which vanishes if (1) $kf^1(y) - c = 0$, and therefore y constant; (2) $k\{f(y) - f(\beta)\} - cy = 0$. The second term of the complete variation, when $kf^1(y) - c = 0$, becomes $\frac{1}{2}\int_{x_0}^{x_1} kf''(y) \,.\, \delta y^2\,dx$; which is negative, since by hypothesis $f''(y)$ is negative for all values of y (with which we are concerned). A maximum, therefore, is afforded by the equations (1) and (2). Combining them with the equation $(x_1 - x_0).y = D$, and eliminating c and y, we find the *extent* of the favoured region, $x_1 - x_0$. Its *position* in the given tract is, as might have been *à priori* expected, indeterminate. Thus the favoured region is *limited* in extent, indeterminate in position; and the law of distribution is equality.

CASE II. Where the sensibility only (β) varies. The first term of the complete variation is the same as in the preceding case, except that (as β now varies) the conditions of its vanishing are now *three*, viz. (1) $kf^1(y) - c = 0$; (2) $kf(y) - kf(\beta)_1 - cy = 0$; (3) $kf(y) - kf(\beta)_0 - cy = 0$. The second term of the complete variation becomes (when $kf^1(y) - c = 0$) half

$$-dx_1^2 k\left[\frac{df(\beta)}{dx}\right]_1 + dx_0^2 \,.\, k\left[\frac{df(\beta)}{dx}\right]_0 + \int_{x_0}^{x_1} \delta y^2 \cdot f''(y) \,.\, dx.$$

By equations (2) and (3) $f(\beta)_1 = f(\beta)_0$. The required region, therefore, comprises either a maximum or a minimum. The latter alternative will generally[2] correspond to the required maximum. ~~Since~~ For as, then, $\left(\frac{df(\beta)}{dx}\right)_1$ is +,

[1] Where δy is an arbitrary variation, *subject to the condition* that

$$\int_{x_0}^{x_1} (y + \delta y)\,dx = D \text{ or } \int_{x_0}^{x_1} \delta y\,dx = 0.$$

[2] What deficiencies there are in this theory may be supplied by Case 4, or by the nature of the functions (continuous, explicit, positive, &c.)

$\left(\frac{df(\beta)}{dx}\right)_0$ is $-$, $\int_{x_0}^{x_1} dy^2 f''(y)\,dx$ being, as in the first case, essentially negative, the second term of the complete variation is necessarily negative. Thus the favoured region is limited, and comprises a region of maximum sensibility; and the law of distribution is equality.

CASE III. Where the capacity only (k) varies. There are two ways of proceeding in problems of this kind, either wholly by the Calculus of Variations (which seems preferable), or as to the form of the function by the Calculus of Variations, and as to the limits by the Differential Calculus exclusively (a method recommended by some—Poisson, I think). The former method was pursued in the preceding cases, but it may be well to give an example of the latter in this case.

First, the form of the function for assumed limits is to be found. It is given by the equation $kf^1(y) - c = 0$, which, since $f''(y)$ is negative (the limits being supposed fixed), corresponds to a maximum [1].

Next, with the aid of the function so found, the limits are to be found by the Differential Calculus from the condition $\int_{x_0}^{x_1} k\{f(y) - f(\beta)\}\,dx = \text{max.}$; where β is constant, k is a function of x, y is a (known) function of x and c, c is a variable, varying with the limits and conditioned by the equation $\int_{x_0}^{x_1} y\,dx = D$.

From the first condition (that the whole is max.),

$$dx_1(kf(y) - kf(\beta))_1 - dx_0(kf(y) - k(\beta))_0 + \int_{x_0}^{x_1} dck f^1(y)\left(\frac{dy}{dc}\right)dx = 0.$$

From the condition (that the distribuend is constant),

$$dx_1 y_1 - dx_0 y_0 + \int_{x_0}^{x_1} dc\frac{dy}{dc}dx = 0.$$

Multiplying the latter by c, and subtracting from the former, we have the equations [2],

(1.) $[k\{f(y) - f(\beta\} - cy]_1 = 0$; (2.) $[k\{f(y) - f(\beta)\} - cy]_0 = 0$.

[1] δy in this method is *quite arbitrary*, c being variable. Contrast Note 1, p. 45.

[2] Since $kf'(y) = C$.

The values of x_1, and x_0, and c, are to be found from these equations, combined with (3) $\int_{x_0}^{x_1} y dx = D$.

Since y involves x only as a function of k, it appears that a solution (*a* solution) of the above system is presented by the solution of the following,

(α) $k_1 = k_0$, (β) either (1) or (2), (γ) (3).

From (α) it appears that the favoured region comprises either a maximum or a minimum. The former alternative corresponds to the required maximum. For the second term of the increment of $\int_{x_0}^{x_1} k\{f(y) - f(\beta)\}$ (subject to the conditions $\int_{x_0}^{x_1} y\, dx =$ given, and $kf^1(y) - c = 0$) is found to be half

$$dx_1^2 \left(\frac{dk}{dx}\right)_1 - dx_0^2 \left(\frac{dk}{dx}\right)_0 + \int_{x_0}^{x_1} dc^2 k \frac{d_2 f}{dy^2} \left(\frac{dy}{dc}\right)^2 dx.$$

The integral portion of this term is by hypothesis essentially negative; and the remaining portion will also be negative, if $\left(\frac{dk}{dx}\right)_0$ is +, $\left(\frac{dk}{dx}\right)_1$ is −, that is, when there is a maximum between k_1 and k_0. Thus, the favoured region is limited, and corresponds to the region of greatest capacity for pleasure; and the law of distribution is such that unto him that hath more capacity for pleasure shall be added more of the means of pleasure.

CASE IV., where both sensibility and capacity (β and k) vary,—

The law of distribution is the same as in the preceding case, viz. $kf^1(y) - c = 0$.

As to the limits, where the co-efficients so vary that for every increase of k there is some increase of π, for all values of y, and *vice versâ*, the following considerations, though not mathematically very elegant, may conveniently be introduced. Suppose k to increase continually as x increases. If any point ξ_1 be assumed for the head of the integral, there can generally be found a lower point ξ_0 for the tail, so as to afford a maximum (ξ_1 being considered fixed). For such a point will be given by the

solution of the equation $\pi_0 - c y_0 = 0$; since the second term of the increment consists (see preceding case), (α) of an integral term essentially negative, (β) of the term $-\frac{1}{2} dx_0^2 \left(\frac{d\pi}{dx}\right)_0$ where it will be observed $\left(\frac{d\pi}{dx}\right)$ is the partial differential with regard to k and β only, not y, and by hypothesis is positive.

If now, the head of the integral, and with it every element, be advanced by one step Δx, by hypothesis the new integral is greater than the old. And, if a new tail is now found for the new head, there is a further increment. And so on: *τῆς ἄνω ὁδου ἀεὶ ἐξόμεθα.* The process might be illustrated by a snake climbing up a straight mountain-path (say in the line of the meridian), re-adjusting his length at each step. If, now, he reaches with his head the top, will he go on? Yes, he will go on over the top of the mountain, until his head is just on a level with his tail. For, if he did not go so far, the end of his tail might with advantage be snipped off, and prefixed to his head; and obversely, if he went further. He will rest in that position; unless, indeed, there be a higher mountain along the meridian. For then it is evident that a portion at least of the snake can be transferred with advantage, so as to cover all the part of the higher mountain which is *above* the horizontal line drawn tangential to the lower mountain. The transferred portion will be taken from the head and the tail. Finally, the new head and tail on the higher mountain will be on a level with each other, and with the new head and tail (if any) on the old mountain. And so on, if there be higher mountains, until a portion at least rests on the highest mountain. But, if there is a ridge stretching *ad infinitum* up to heaven, then the snake will get up on that ridge as high above the highest mountain as he can.

These considerations may complete, as indeed they might have constructed, the reasoning about the limits in the previous cases.

It is not pretended that even the mathematical reasoning of the preceding theory is free from objection. Thus,

it might be objected that account is taken only of *some* solutions of the system of equations for the limits in the third case of the second problem. But the considerations indicated under the fourth case are, I think, of general application. Again, it might be objected that a *maximum* found by the Calculus of Variations, even *the greatest* maximum so found, may not be the *greatest possible* value of a proposed integral. But in the present case, when the integral is at its maximum, the first term of its variation is zero, and the second term is negative. And however the integral begins and continues varying from that maximum, by the variation of the function only (limits remaining fixed), the second term of its variation is continually negative. For it is,

$$\tfrac{1}{2}\int_{x_0}^{x_1} f''(y)\,\delta y^2\,dx$$

where $f''(y)$ is negative for all values of y. Therefore, the first term of the variation is continually negative. Therefore the integral, however and how much soever it varies from the maximum, continually decreases. Therefore the maximum is the greatest possible value. And if the somewhat amphibious investigation emerging from the Calculus of Variations[1] takes its stand on the Differential Calculus alone, the same result is even more apparent. If the limits vary, these considerations, combined with those indicated under the fourth case, suffice to obtain the greatest possible value of the integral. Again, it may be objected that the equation $kf'(y) - c$ may afford more than one available value for y; that the solution may be *discontinuous* (as if there should be one law of distribution for brutes, another for men); and that in such a case the law of privilege might no longer prevail. But, since π is an explicit continuous function of y, and since $\left(\frac{d_2\pi}{dy^2}\right)\delta y^2$ is continually negative, it is evident that a certain value of $\left(\frac{d\pi}{dy}\right)$ produced by a certain value of y, can never be

[1] Supra, p. 44.

reproduced by any other value of y. It might be otherwise, if the condition $\left(\frac{d_2\pi}{dy^2}\right)$ to be negative were even slightly infringed. Let $\pi = F(xy)$[1] where $\frac{dF}{dx}$ is continually positive; a form to which the essential part of the reasoning in the problems applies. Let this surface, generally concave, become convex for a space (say along a line parallel to the plane πx), then, suffering a second inflexion, resume its concavity[2]. Then there might be two values of y, $\theta_1(xc)$ and $\theta_2(xc)$ satisfying the equation $\left(\frac{d\pi}{dy}\right) = c$. If we could be sure that the convex part of the surface was not available for the greatest possible value of the integral, then the greatest possible is afforded by either of the above curves, or by some combination of both. The law of privilege prevails as between the elements of each curve, but not as between the curves[3].

Subject to the mathematical difficulties (if any), the preceding theory is unaffected by several modifications of the data more agreeable to the subtlety of nature than the simplicity of a first statement.

(1.) If k and β are functions of *several* variables (as the co-efficients of pleasure well may be), analogous reasoning holds good (as, indeed, is *à priori* evident). To take, for example, the most difficult cases, the third and fourth cases of the second problem.

Let the integral be,

$$\int_{{}_1x_0}^{x_1}\int_{{}_2x_0}^{{}_2x_1}\cdots\int_{{}_nx_0}^{x_1}\int_{y_0}^{y_1}\left[\pi - cz\right]d_1x\,d_2x\ldots d_nx\,dy$$

where $\pi = k\{f(z) - f(\beta)\}$; k is a function of the variables

[1] Infra, p. 57.

[2] It is not necessary to suppose a violation of the condition $\left(\frac{d\pi}{dy}\right)$ to be positive, for the values with which we are concerned. This condition, by the way, though doubtless generally realized, is not essential to the *general* reasoning. The essential condition is $\left(\frac{d_2\pi}{dx\,dy}\right)$ positive. Cf. Infra, p. 57, 53.

[3] Infra, p. 62.

${}_1x, {}_1x, \ldots {}_nx, y$; z is the stimulus to be assigned; and there is sought for the limit of integration a closed function—closed figure in space or hyper space—$F({}_1x\,{}_2x \ldots {}_nx\,y)$ (whereof, equated to 0, y_1 and y_0 are to be regarded as two roots in terms of the other variables). Then, by the principles laid down in works on the Calculus of Variations [1], the first term of the complete variation is,

$$\int_{{}_1x_0}^{{}_1x_1} \ldots \int_{y_0}^{y_1} \left[k f^1(z) - c \right] \delta z\, d_1x \ldots dy,$$

$$+ \int_{{}_1x_0}^{{}_1x_1} \ldots \int_{{}_nx_0}^{{}_nx_1} \left[\underset{y_1}{(\pi - cz)}\ \delta y_1 - \underset{y_0}{(\pi - cz)}\ \delta y_0 \right] d_1x - d_nx.$$

where δz is an arbitrary variation of z, subject only to the condition that $\int \int \ldots \int z\, d_1x\, d_2x \ldots dy = D$; $\delta y_1\ \delta y_0$ are arbitrary variations of $y_1\ y_0$; $(\pi - cz)_1$ denotes that in this expression there is substituted (first, for z its value in terms of the other variables, and then) for y its value y_1 in terms of the other remaining variables. The form of z is given by the equation $kf^1(z) - c = 0$. The form of the limiting function is $\pi - cz = 0$. These conditions afford a maximum when the delimited region comprises a maximum of k. For the second term of the complete variation is seen to consist partly of an integral term which is essentially negative, if $f''(z)$ is negative; and partly of a set of integral terms (of an order of integration less by one than the preceding), which are negative generally, if a maximum of k be comprised. For example, in the case of two variables (putting $f(z)$ for $f(z) - f(\beta)$) half

$$\int_{x_0}^{x_1} \int_{y_0}^{y_1} \delta z^2\, k f''(z)\, dx\, dy.$$

$$+ \int_{x_0}^{x_1} \left[\delta {y_1}^2 \left(\frac{dk}{dy} f(z) \right)_{y_1} - \delta {y_0}^2 \left(\frac{dk}{dy} f(z) \right)_{y_0} \right.$$

$$+ d{x_1}^2 \left[\frac{d}{dx} \int_{y_0}^{y_1} \{ k f(z) - cz \}\, dy \right]_{x_1}$$

$$- d{x_0}^2 \left[\frac{d}{dx} \int_{y_0}^{y_1} \{ k f(z) - cz \}\, dy \right]_{x_0}$$

[1] Jellett, Todhunter, Strauch, &c.

The latter terms are seen to be negative when the favoured region is concave with regard to the plane *xy*. The *skirt* of the integral is held up at all points to the same height (given by an equation of form $\psi(k) = 0$).

Generally, in the third and fourth (and other) cases of the second problem, the considerations before placed under the fourth case are equally applicable, *mutatis mutandis*, to functions of many variables. Varying the snake-metaphor, we may now regard all living beings congregated on the mountain-tops; the highest and most favoured regions being occupied by the most capable. The less qualified are relegated to the skirts of existence, which consist of horizontal curves along the mountain-sides, curves all of the same altitude. But, if there is anywhere a mountain-ridge stretching infinitely up to heaven, thither all creation is to tend, the most gifted in the van.

To avoid confusion between the thing compared and that with which it is compared, it is proper to observe, that each of these metaphorical mountaineers corresponds *strictly*, not to an individual sentient, but a sentient element (supra, p. 41). The greatest quantity of stimulus is assigned not to the sentient individual who owns the *highest* sentient elements, but to him who owns *most highest* elements (in the sense of double maximum above explained, genus 2, species 2, supra, VII.). The individual who owns the very apex of the topmost mountain *may* possess no other tract at all within the favoured region, within the region which is, so to speak, taken into cultivation. He will not then, because he has a very small tract of the most fertile ground, obtain from the common store more seed than he who has a very large tract of averagely good ground half-way down the mountain.

(2.) The reasoning of the problems holds good, if the distribuend is regarded not as constant, but as varying with the extent of the region over which the distribution is made; so varying, that for every increase in the extent of the region, there is *some* increase in the distribuend, but the rate of the distribuend's increase decreases as the extent of region increases. These conditions would be ex-

pressed by putting $D = \phi(x_1 - x_0)$ [1], where ϕ' is always + positive and ϕ'' always negative. To excite interest in this case, it may be well to make the premature remark, that these are precisely the relations which Malthus correctly supposed to exist between the quantity of food and the number of population, and which he illustrated by the properties of the logarithm. In this case, then, we must have $\int_{x_0}^{x_1} \left[k(f(y) - f(\beta)) - cy\right] dx + c\,\phi(x_1 - x_0)$ a max. The form of the required function is the same as before. The limits are given by the equations,

$$k_1\{f(y_1) - f(\beta)\} - cy_1 + c \times \phi'(x_1 - x_0) \times \phi'(x_1 - x_0) = 0$$

$$k_0\{f(y_0) - f(\beta)\} - cy_0 + c \times \phi'(x_1 - x_0) \times \phi'(x_1 - x_0) = 0,$$

combined with the equation $\int_{x_0}^{x_1} y\,dx = \phi(x_1 - x^0)$. The fulfilment of these conditions corresponds in the same cases as before to a maximum. For, in fact, the only new terms added to the second term of the complete variation are, $\frac{1}{2}\, c\, \phi''\, (x_1 - x_0)\, (dx_1 - dx_0)^2$, which by hypothesis is negative. The snake-argument is unaffected.

(3.) By a parity of reasoning, the same conclusions are obtained if the distribuend varies not only with the *extent*, but the *position* of the favoured region, with the extent as before, and with the position such that an increase of capacity over the whole region is attended with an increase of the distribuend, but not so great an increase as to destroy the first condition. (There will at once occur to the reader the analogy of the quantity of food increasing, not merely with the number, but with the quality, inventive power, industry, &c., of population, but not ultimately so increasing as to vitiate the essential position of Malthus.) In this case the distribuend might be viewed as an integral between the limits x_1 and x_0, say $\psi(x_1) - \psi(x_0)$: such that $\psi^1(x_1)$ is +, $\psi^1(x_0)$ is −, $\psi''(x_1)$ is −,

[1] Using only one variable for the sake of brevity, though of course the method is quite general; and considering, *exempli gratia*, Problem ii. case 3.

$\psi''(x_0)$ is $+$: in which case there is a parity of reasoning. The snake-argument becomes *à fortiori*.

It would be possible securely to superimpose many other imaginable modifications on the preceding theory, which seems to be rigorously true of the pleasures of isolated sensations. But, whatever might be the practical value of such deductions, they would not have here much theoretical interest; inasmuch as, while limited to isolated sensations, they would afford only a slight (though a real) verification of exact utilitarianism. But it would be far otherwise if we were warranted in substituting in the preceding problems for sensory elements, sentient individuals, and for the indeterminate "capacity of pleasure," the more definite conception of "quantity of emotion[1]," which Herbert Spencer has shewn to be an incident of evolution, or rather, substituting not the incident but the differentia itself of evolution; whatever may be its most appropriate conception[2], most probably some quantity of motion. We should then have a series of hypothetical propositions which are strikingly consilient with common sense or common utilitarianism. It will conduce to clearness of conception first to enounce these propositions in all their unqualified breadth, then to examine their evidence and the qualifications it imposes, exhibiting at each stage the ethical importance of the deductions.

Problem I. may now be worded: Given a certain amount of distribuend (wealth, power, &c., strictly speaking, means to stimulus), to be distributed among a given set of sentients (upon condition that each shall get some share) to find the law of distribution. Case I. and II., where the sentients do not differ in the order of evolution. The law of distribution is equality. This deduction agrees with the principle of common sense (noticed by Mr. Sidgwick), that where no reason to the contrary can be shewn, there

[1] The reader is requested to make allowances and corrections, if the language has already appeared hovering, preparing for this hypothetical flight above the region of sensation.

[2] Herbert Spencer, "Psychol." i. § 215, chap. I. b. First Principles, &c.

is a presumption in favour of equality; with the doctrine of Bentham, "everybody to count for one," in spite of (Case II.) *some* differences. It is submitted that (contrary to the statement of Mill, "Utilitarianism," p. 92) this principle *is* "a logical corollary" (of exact utilitarianism), and is *not* "involved in the very meaning of utility." Cases III. and IV., where the sentients differ in the order of evolution. Unto him that hath higher development shall be added more of this world's goods. This deduction agrees with common sense, as exhibited in the approved dealings of men with animals, of civilized with savage races, in the privileges of aristocracy approved in ages when aristocracies really represent a higher order of evolution. The deduction also agrees sufficiently with utilitarianism. For it was never intended by a sound utilitarian that "Mr. Pongo" was to count for one (cf. Mill, "Utilitarianism," p. 93). It is true that as between men utilitarians usually presume that the law of distribution is equality; either because any other distribution would be attended with preponderant disadvantages,—the pangs of envy, sense of injustice, &c. (which, however, on the utilitarian theory of justice would not have place);—or rather, because they take for granted that there is no *material* difference (no difference of kind, as Mill says in his "Logic") between human creatures. If, however, utilitarians were really convinced that there existed either now, or (what is more conceivable) in a past stage of the world's evolution, a broad distinction such as that which is indicated by Aristotle's theory of the φύσει δοῦλος, presumably the establishment of a privileged class would commend itself to utilitarian sense, as well as to the more metaphysical, yet essentially practical, genius of Aristotle.

If we now draw the line between the Problems[1] rather differently than was convenient for the purposes of mathematical illustration, Problem II. becomes: The distribuend being given, or varying with the number and evolution of the distributees in the manner in which wealth

[1] The references in the sequel are to this arrangement.

is known to increase with the number and civilization of population, to find the number and nature of the distributees. The most important application of the problem appears to be to the theory of population. All the cases shew that the number is limited, agreeably to the Malthusian doctrine of our utilitarian economists. The last three cases point to the qualitative improvement, as well as the quantitative limit, of population. Both those doctrines are utilitarian. The latter, indeed, has been more fully discussed by utilitarian professors; and for a bold presentation of the former, we might have to look away to the soaring utilitarianism of Plato, when he taught, *καλλιστὰ γὰρ δὴ τοῦτο λέγεται καὶ λελέξεται ὁτὶ τὸ μὲν ὠφελιμὸν καλὸν τὸ δὲ βλαβερὸν αἰσχρὸν*[1]. But as Plato did not ignore the latter principle[2], so presumably the modern utilitarian would accept the former principle, (though he may hold that the practical applications proposed, e.g. by Plato, are both inefficacious, and attended with preponderant disadvantages). In fact, we have seen that Mill's ideal is a society not only *limited*, but *cultivated*.

The preceding conclusions are deducible by reasoning analogous to that which was employed in the simple cases of isolated sensations; subject to the same mathematical difficulties, and given the following analogous conditions.

(I.) The pleasure of an individual sentient must be capable of being regarded (under normal circumstances, *ceteris paribus, abstractis abstrahendis*, upon a sufficiently large scale to eliminate chance, with sufficient accuracy for practical purposes, &c.) as a function both of the nature of the individual—certain co-efficients denoting the degrees in which the individual possesses certain qualities, varying with different individuals,—and also of the stimulus applied to the sensory organs of the sentient; the stimulus involving the individual's share of the distribuend (all the material goods or means to stimulus which are capable of being distributed among sentients), but not

[1] "Republic," v.

[2] As Malthus points out. He does not notice, "Repub." ii. *οὐχ ὑπὲρ τὴν οὐσίαν ποιούμενοι τοὺς παῖδας εὐλαβούμενοι πενίαν.*

involving any variable co-efficients: e.g. $\pi = f(\beta, k, \ldots \sigma)$; where $\beta k \ldots$ are variable co-efficients, $\frac{1}{\beta}$ is the sensibility as in the first investigation, k has the second intention of order of evolution (supra, p. 54) . . . , σ is a function of $y, = \phi(y)$, σ being the stimulus, and y the share of distribuend.

(II.) $\left(\frac{d_2 \pi}{d y_2}\right)$ must be negative for all values of y, and of the co-efficients, (or all with which we are concerned): or, at any rate, there must exist satisfactory criteria of a maximum.

(III.) The co-efficients must be capable of being regarded as so interdependent that the differential (complete with regard to k, partial with regard to y) $\frac{d}{dk}\left(\frac{d\pi}{dy}\right)$ is continually positive. If each of the co-efficients be regarded as a function of a new variable x, this condition may conveniently be expressed:

$\frac{d}{dx}\left(\frac{d\pi}{dy}\right)$ always +, $\frac{dk}{dx}$ being + (always −, $\frac{dk}{dx}$ being −).

(4.) With the same notation, $\frac{d\pi}{dx}$ +, $\frac{dk}{dx}$ being +.

None of these conditions are fulfilled; and even if they were fulfilled, the mathematical reasoning which starts from them, though apparently secure, is led through a region abounding in crevasses. Accordingly, the conclusions begin to look very small. It is worth while, however, to examine the degrees of invalidity in the different links of our chain, to see if there is sufficient coherence in any part of it to support any conclusion favourable to exact utilitarianism. How far then are the preceding conditions fulfilled, or, if not fulfilled, not indispensable? In answer to these questions, a few unfinished remarks will here be offered on each condition separately, so far as practicable.

(1.) The first condition is not strictly fulfilled, even in the region of sensation. For it is obvious,—though it may seem not to have been noticed by some who have

spoken hastily about the relation of "fortune physique" to "fortune morale"—that the stimulus applied to the sentient *does* depend upon, and involve his nature, as well as his means. With the same amount of material means, of any such means as are capable of being distributed, the more gifted individual secures to himself a greater amount of those tactile impressions, those odorous and gustatory influences, those aery and etherial undulations, which constitute the "stimulus," or "Reiz." With the same means of making an excursion, the "boy with eyes" obtains more sense-awakening impressions than "the boy without eyes." But the theory is not materially affected by this modification; since it is probable that the power to extricate more stimulus from the same means, co-exists with the power to extricate more pleasure from the same stimulus. The pleasure, though a more complicated and less precisely ascertainable function of the co-efficients and the share, may still be such a function of them as the theory requires; say, $\pi = f(\beta k \ldots \phi(\beta x \ldots y)) = F(\beta k \ldots y)$, where F fulfils the three latter conditions. But further, as we consider the sources of pleasurable stimulus—*ὁπόθεν ἡ πρὸς ὄψιν ἡ πρὸς ἀκόην τι προσβάλῃ ὥσπερ αὔρα*—we find for each, at least each human sentient, one paramount such source, his fellow-creatures. And the emanations received by him from this source depend not only on the nature[1] of his associates, but also upon their means, *their* shares of the distribuend. The very mildest mathematical expression applicable to the integral of the quantity of pleasure which each social unit thus receives from his society is $\int_{x_0}^{x_1}\left[F(\beta k..y\int_{\theta}^{\eta} f(\beta k \ldots y)\,dx\right]dx$;

[1] Of the *useful* qualities account has already been taken in the enunciation of the second problem (cf. p. 53). They go to swell the distribuend. Such are physical strength, inventiveness, industry, ambition as the spur to useful energy. It is difficult to demarcate these tributaries of the distribuend from the qualities which are now under consideration, as immediate sources of pleasurable stimulus, the qualities *directly pleasurable* to others—ascending from unsympathetic instinct, through the whole scale of more and more symphonious passion, to the qualities which irradiate the purest pleasures, to rational benevolence, and romantic love, and friendship, and even friendliness and good manners.

where the left-hand symbols denote the nature and share of the individual, and the integral within the integral denotes the stimulus which the social unit centring receives from his associates, a stimulus integrated over appropriate limits, which vary with the individual's order of evolution (η and θ functions of x). Or this formula,

$$\int_{x_0}^{x_1} \pi \, dx = \int_{x_0}^{x_1} \left[\int_{\theta}^{\eta} f(\underline{xy}\, xy) \, dx \right] dx;$$

where the left-hand x and y underlined are the nature and share of the social unit; the right hand x (in turn) and y denote the nature and share of each of his associates; f denotes the quantity of pleasure resulting to the individual from the interaction of his own nature and that of each of his associates, depending in some degree upon the means of each. Accordingly, in the first integration the underlined x and y remain otiose. The first integration extends over appropriate limits expressed as functions of x (perhaps for some natures over the whole area of sentience x_1 to x_0). The second integration makes no distinction of x. The quantity of the pleasure of "all sentients" is found by adding together the pleasures of each; which, by the first integration, are represented as functions of the share and nature of each; or perhaps, if the underlined $\underline{y}$ were omitted in the latter formula, the social pleasures of the individual would be sufficiently represented by the interior integral. If, then, $\left(\frac{d}{dy}\right) f(x\, xy)$ increases with the increase of *either* x, as is reasonable to suppose; if the extent of the interior integration increases with the underlined $\underline{x}$, corresponding to the wider sympathies of higher natures; and, if $\left(\frac{d_2 f}{dy^2}\right)$ is negative; then, by changing the order of the integration, it may be deduced that $\frac{dy}{dx}$ is positive. There is deduced the privilege of the gifted; whereof a particular case is, that equal should be distributed to equals.

It may be objected that these conclusions are ob-

vious to common sense; that the Mathesis of Hedonism, when we

"deduct what is but vanity or dress,"

is insignificant. But it is submitted in reply, first, that the mathematical deduction from a proposed general practical principle of received particular (or less general) principles, tends in the logic of practice to confirm the general principle; very much as in the logic of science, the mathematical deduction of observed facts from hypothetical theory confirms the hypothesis; for instance, in the case of the undulatory theory of optics. The analogy must not, indeed, be pressed too far. For, in the case of the physical hypothesis, the observations are delicate, the deductions profound; while for mathematical ethics, an elementary knowledge of the calculus, and the experience of common sense, suffice. But this difference may be considered accidental, depriving the demonstrator, indeed, of credit, but not the demonstration of credibility. Nor, again, in the case of mathematical ethics, is there that nice and manifold consilience of fact with theory which in mathematical physics, notwithstanding some dark utterances of Mill[1], seems of primary importance in confirming hypotheses. It must also be admitted, that the essential feature of the analogy is somewhat dim; that the passage from common sense to an "unconsciously implicit" general principle, requires illumination. Some remarks upon this topic have already been ventured, though on a topic so obscure it might be wiser to have imitated the logical sobriety of the author of the "Methods." It is further submitted in defence of this method, that when we have, to adopt a Humian metaphor, laboriously marched upon the capital through and with the aid of the majority of the provinces, we may then be in a position to explore and reduce outlying regions, whose existence was before unknown, or whose dependence doubtful. Having attained the general principle by a copious induction, we are able to deduce particulars which were previously un-

[1] On hypotheses.

observed, or which were regarded as exceptions to the less general (the empirical) laws. For so far are the previous deductions from being self-evident truisms, that they are not even universally true. We have already had an instance[1], in which, in spite of difference in sensibility, the most felicific distribution is, not privilege, but equality; the rationale of which instance is not, perhaps, immediately obvious to unsymbolic sense. And generally, all the previous reasonings depend upon the sign of the second term of variation; an investigation about which it may be pretty safely said, that the ἀναποδεικτοὶ γνῶμαι of common sense are not very valuable. If the second term were positive, it would not be true that the most felicific distribution among equals is equality; "anything but that," that would be the *most infelicific* distribution. It is not, indeed, supposable that the second term could be continually positive; but it is not only supposable, but possible, and even probable,—being agreeable to the most scientific exposition of the subject which we possess, the pleasure-curve of Wundt,—that $\left(\frac{d_2\pi}{dy^2}\right)$ is positive for the *lower* values of y above the threshold of pleasure. Accordingly in the "koomposh" of an unlimited pauper population, the most favourable disposition might seem to be (abstracted from practical considerations, and *if* the delineation of Wundt be verified within and beyond the region of sensation), might seem perhaps to be, that adhering *ex hypothesi* to the letter of the first problem, we should be guided by the spirit of the second problem, should wish to cut off the redundant numbers with an illusory portion, so as to transfer substantial (equal) portions to a few. There might be, as it were, a mulcting of many brothers to make a few eldest sons. The given materials would not be built into an edifice of uniform height (the *worst* arrangement now), but into a low wall surmounted by lofty towers[2]. Or again, if the pleasure-

[1] Case 2, Problem I.

[2] Whether the resemblance of certain semi-civilized societies to this figure is merely an ironical accident, or rather an instance of that first approximation

surface should present a convexity between two concavities, as in a case already considered [1], it might happen that the most felicific distribution among equals would be unequal. Nay, so illusory is the nature of things, that equality in this case might afford a *maximum*, but inequality the *greatest possible quantity*, of pleasure [2]. The privilege of the gifted also depends upon the sign of the second term, which being changed, what was before the most felicific becomes now the most unfelicific arrangement. "Corruptio optimi fit pessima." So, again, even where a maximum greatest quantity was afforded, the law of privilege might be violated. If the pleasure-surface consisted of two billows swelling concavely to the plane of axes x and y, and separated by a convex trough, as in the case already considered, the higher billow which was further from the abscissa almost uniform in the direction of x, and barely fulfilling the conditions $\left(\frac{d\pi}{dx}\right)\left(\frac{d_2\pi}{dy\,dx}\right)\not>$, positive, while the nearer and lower billow rôse steeply in the direction of x; then, should we have to *tack* [3] mathematically over these billows in order to reach the utilitarian end, it would generally happen that the ys of the upper curve would be assigned to the shorter xs, the largest portions to the individuals least qualified to enjoy. For, if possible, let the obverse arrangement correspond to the greatest possible value. Then a greater value could be obtained by reversing that obverse arrangement. For, by that reversal, the ys of the further curve would not materially lose, while the ys of the nearer curve would materially gain. The utilitarian method, then, is not necessarily plain sailing.

which (in spite of Mill) the natural and actual often makes to the utilitarian ideal, might be curious to enquire.

[1] Supra, p. 49.

[2] To be sure it may be said, that this latter kind of inequality is only theoretical, for that in practice it would be desirable, by a continual *tacking* between the upper and the lower curves of distribution, to give each individual shares in both dispensations, and so a compound equal share. If the theory does not forbid this compromise, at any rate it does not enjoin it.

[3] Todhunter, "Researches in Calculus of Variations."

The enquiry, therefore, not being otiose, let us proceed to investigate more accurate formulæ. Let there be an ideal society, homogeneous as to tastes and pleasures; then, whatever be the nature of their sympathetic pleasures,—in parties and banquets, in games and excursions, conversations and debates; in colleges, in clubs, in families; through every variety of *φιλία*, *φιλότης*, *εὐτραπελία*; fellow-students and partners, brothers and lovers,—that equal should be distributed to equals is mathematically deducible from exact utilitarianism. For let the self-regarding pleasures of an individual, suppose number one, be

$$\alpha_{11}(a_{11}) + \alpha_{21}(a_{21}) + \alpha_{31}(a_{31}) + \&c.$$

where the Greek letters denote the forms of functions, the first subscribed number denoting the nature of the function, the second the individual under consideration. The corresponding Roman letters express the amount of means (wealth, &c.) expended on each pleasure. Then by hypothesis $\alpha_{11} = \alpha_{12} = \alpha_{13}$, &c., $\alpha_{21} = \alpha_{22}$, &c. (equals as touching the form of the function, is identical). Let $\beta_{11}(b_{11} + b_{12}) + \beta_{12}(b_{11} + b_{12})$ denote the sum of the sympathetic pleasures of a couple of individuals, upon whose mutual pleasure has been expended the amount $b_{11} + b_{12}$. Then by hypothesis the pleasures of each are equal. By hypothesis also $\beta_{11} = \beta_{12} = \beta_{13} = \&c.$ (as to the form of the function). Let there be a similar notation for triplets, and more extended social circles. Then if each of the Roman symbols, each of the many expenditures made for each individual, be regarded as arbitrary, independently variable, subject only to the condition that the sum of all the expenditures should be equal to a constant quantity (the distribuend), the utilitarian problem would become to find $a_{11}\ a_{12}$, &c., b_{11}, &c., such as to afford the greatest possible value of the following expression,—

$$\alpha_1(a_{11}) + \alpha_1(a_{12}) + \&c. + \alpha_2(a_{21}) + \&c.$$
$$+ 2\beta_1(b_{11} + b_{12}) + 2\beta_1(b_{13} + b_{14}) + \&c. + 2\beta_2(b_{21} + b_{22}) + \&c.$$
$$+ 3\gamma_1(g_{11} + g_{12} + g_{13}) + \&c.$$
$$- c(a_{11} + a_{12} + \&c. + a_{21} + \&c. + b_{11} + b_{12} + \&c. + g_{11} + \&c.),$$

where c is an arbitrary constant, to be properly determined.

Differentiating and equating the first term of the increment to zero, we have the following equations,—

$a'_1(a_{11}) - c = 0$; $a'_1(a_{12}) - c = 0$, &c.

$2\beta'_1(b_{11} + b_{12}) - c = 0$; $2\beta'_1(b_{13} + b_{14}) - c = 0$, &c.

$a_{11} + a_{12} +$ &c. $+ \Sigma b + \Sigma g +$ &c. $= D$.

The second differential of each of the functions being continually negative, the greatest possible value is afforded by the roots of these equations; and each equation has only one available root.

Therefore, $a_{11} = a_{12} = a_{13}$, &c.

$(b_{11} + b_{12}) = (b_{13} + b_{14})$, &c.

$(g_{11} + g_{12} + g_{13}) = (g_{14} + g_{15} + g_{16})$, &c.

Thus equal amounts are expended; on each individual, for his self-regarding pleasures; on each couple, for their *egoisme à deux;* on each triplet, and more numerous association, for their social pleasures. The theory does not pronounce upon the manner in which the distribuend should thus be equally applied; whether, for example, the expenditure of a household should be defrayed directly out of the common store, or should pass through the hands of the members of the household. The theory only pronounces that the expenditure of each (similar) household should be the same.

Let the ideal society be so far heterogeneous as to be divisible into two classes, such that each member of one class enters into an intimate and exclusive agreeable relation with a member of the other class. The classes being designated by odd and even numbers respectively, let the hedonistical symbol of a couple be $\beta_{11}(b_{11} + b_{12}) + \beta_{12}(b_{11} + b_{12})$; where β_{11} is not, by hypothesis, equal to β_{12}; but β_{11} is identical with β_{13}, β_{15}, &c., β_{12} identical with β_{14}, β_{16}, &c. Then the terms of the increment, corresponding to these peculiar relations, are,—

$(db_{11} + db_{12})([\beta'_{11} + \beta'_{12}](b_{11} + b_{12}) - c)$

$+(db_{13} + db_{14})([\beta'_{11} + \beta'_{12}](b_{13} + b_{12}) - c)$, &c.

(where $[\beta'_{11} + \beta'_{12}]$ denotes a function).

Hence it is deduced that the material provision for each of the couples, the *χορηγία* of each concord, shall be the same.

Apart from such intimate relations, suppose a judicious intermixture of odd and even numbers to lend such agreeable animation to their mutual society, that the odd and even hedonistical symbols are not by hypothesis to be assumed as identical. Then for every such party, of twenty-four for example, we have the equation, $12\,[\omega'_1+\omega'_2]\Sigma o-c=0$; whereof only one root is available. Therefore, the cost of every such festivity throughout the ideal society shall be the same. If in practice the expenditure on such an occasion was directed to procuring pleasures of the α class also (e.g. that of supper), or also of the β and the γ species, the argument would not be affected.

By a parity of reasoning, the same sort of equal distribution is deducible in the case of other pleasures (perhaps the literary and artistic), which, though not necessarily equal, are mutual, in respect to which an ideal society might so far be homogeneous, that persons whose participation is essential to the sympathetic pleasure, are to be found indifferently in all sections of the society. *Primâ facie* a very witty man ought perhaps to have more means of enjoying his wit (e.g. more respite from the average amount of work expended upon the distribuend). But if he could only enjoy his wit in the company of an (appreciative though not necessarily witty) audience, this *primâ facie* claim to privilege would disappear. Where, however, there exists a society within a society, who, by an exclusive intercourse *inter se*, are capable (in virtue of a higher nature) of deriving a more exquisite pleasure, than by association with an inferior class, there begins to arise a genuine case of privilege; as in the relation of men to apes, and perhaps of the civilized to the savage.

The case of privilege may abstractedly be represented by the symbols just employed. Thus let a_{11} (a_{11}) be the self-regarding pleasure of Robinson Crusoe; a_{12} (a_{12}) that of Friday. Let their mutual pleasure be $[\beta_{11}+\beta_{12}]$ $(b_{11}+b_{12})$. Then, by the usual reasoning, we have $\alpha'_{11}(a_{11})=c$, $\alpha'_{12}(a_{12})=c$, $[\beta'_{11}+\beta'_{12}]$ $(b_{11}+b_{12})=c$.

Now, Robinson Crusoe, in virtue of his superior nature, his "higher intellectual," and greater quantity of emotion,

for the same increment of external means, procures a greater increment of happiness. For the same value of the variables, $a'_{11} > a'_{12}$. Also, the second differentials of both functions are continually negative. And from these two properties, combined with the condition above found, $a'_{11}(a_{11}) = a'_{12}(a_{12})$, it readily follows that $a_{11} > a_{12}$; Robinson Crusoe's private portion exceeds Friday's. It may be observed that the theory (unlike the story, I think,) leaves open the question whether Robinson Crusoe takes more pleasure in the society of Friday, or Friday in that of Robinson Crusoe.

Of course, it is not pretended that formulæ so abstract could be usefully applied to actual inequalities, even when they naturally exist, much less when they are artificially imposed. Merely dipping into the concrete, and long before we are "immersed in matter," we may meet with formulæ which, though still abstract, and arbitrarily simplified[1], and quite inadequate to express the sumless pleasures with which they deal, do not, even thus, even theoretically lend a very legible law of distribution. It is possible, however, to exaggerate the deficiencies of calculus. Thus, it has often been triumphantly asked, With what success could mathematical calculation address itself to social phenomena, when it is unable to cope with the problem of three bodies. But perhaps the example of mechanics might suggest another conclusion, namely, that mathematics are capable of advancing victoriously, even while leaving impregnable fortresses in the rear. And so in the class of problems before us, even when they are most uninviting, it might be hoped that approximative methods would be attainable, if a sufficiently clear and appropriate conception of the data were obtained. In the case of the last of the formulæ suggested at p. 58, suppose one of the conditions, that it would be impossible to secure a greater inequality of distribution than that represented

[1] In particular, political considerations are to be relegated to a rather comprehensive class of "other things being equal."

by a given curve y[1]. Then y can only vary from this given curve so that for each element the new $\frac{dy}{dx}$ should be less than the old (supposed continually +) so that $\frac{d}{dx}\delta y$ should be always negative. Suppose the distribuend, the limits x_1 and x_0, and the values of y at the limits x_1 and x_0 to be fixed. Let the interior integral also have x_1 x_0 for limits. Then, by proper manipulation, the first term of the variation from the given curve can be expressed,—

$$\int_{x_0}^{x_1} R \frac{d}{dx} \delta y - \int_{x_0}^{x_1} S \times \int_{x_0}^{x_1} T \times \frac{d}{dx} \delta y,$$

where R, S, T are known functions (derivable from the given curve). *If*, then, R be continually (for all values of x between the limits) positive, while S and T are continually of opposite sign, the first term of the variation is essentially negative. And[1], therefore, the given curve affords a maximum of the integral, which maximum *may be* the greatest possible value[2]. Or again, if we might suppose, what is more appropriately to be supposed, that no one's pleasure in the way of social intercourse is appreciably affected except by those between whom and himself, and between whom themselves, there were to be wished equality of distribution[3], we might be warranted in writing

$$\int \pi \, dx = \int F(xy f(xy)) \, dx,$$

where the nature and material advantages are treated as constant throughout each social area. Or rather, not making so violent a supposition,

$$\int \pi \, dz = \int_{2_0}^{2_1} \int_{\theta}^{\eta} \int_{\theta}^{\eta} f(\underline{\underline{y}} \, \underline{x} \, x) \, dx \, dx \, dz,$$

where $\underline{x}$ is otiose during the first integration, $\underline{\underline{y}}$ during the first two integrations. Here all sentients may be supposed to be arranged,

"Beast, bird, fish, insect," graduate up to man and

[1] Todhunter, "Researches in Calculus of Variations."

[2] This method is applicable to much more complicated convolutions.

[3] Cf. Mill, "Utilitarianism," p. 47.

the higher races of man, by steps not infinitesimal but finite. (The external symbol of summation should therefore rather have been Σ than $\int$). The internal summation (where the number of the units is likely to deserve the symbol $\int$) is performed over the whole of each step, but not further. The limits, the areas of the steps, may be treated as given for each height on the "world's great altar-stair" (first problem), or as variable (second problem), and partly interdependent, e.g. a number of higher sentients requiring a number of lower sentients for food and service. Y is constant for each step, a condition not mathematically justified, but empirically imposed. Here, then, if π fulfil the second and third[1] conditions (p. 57), the theory as usual points to a law of privilege. The inhabitants of each step are on an equality with each other, privileged above the inhabitants of inferior steps. Thus the theory seems apposite to the cases where there is a great break in the order of evolution, as between man and beast.

Before dismissing the consideration of the more complicated formulæ, it is important to notice that it is in general theoretically possible, when the *form* of y is given or assumed, to find values of the *limits* such that the first term of the increment of $\int \pi\, dx$ should vanish; and that presumably some one of these solutions corresponds to the greatest possible value of the integral.

(II.) To prove that $\frac{d_2 \pi}{dy^2}$ is negative, it is possible to cite the authority of Laplace and others, who have described "fortune morale" as the logarithm of "fortune physique;" unless, indeed, they may seem to designate by "fortune morale" rather the pleasures of the possessor than those of the consumer. ("Panis, ematur, olus," &c.)

However that may be, it is possible to cite the testimony of common experience, thus summed up by Mr. Sidgwick for instance, after a careful examination of common sense:—

[1] Here that $\frac{d_2 \pi}{dx dy} = +$.

"We may conclude that, while the richer man is on the average happier, yet increase of happiness attends increase of wealth in a rapidly decreasing ratio [1]."

There seems also to be some weight in the following considerations. In the integrals of means and pleasure, consider any two pair of elements, proportional to y', $F(x'y')$ and y'', $F(x''y'')$; as it were two parallelopipedal material wicks, each with a similarly-shaped flame of pleasure. Varying the length of the wicks, while preserving the sum of the lengths constant, determine the lengths so that there should be a maximum or minimum of joint-flame. If $\frac{d_2 F}{dy^2}$ is always positive, there will be a minimum when $\frac{dF(x'y')}{dy'} = c$, $\frac{dF(x''y'')}{dy''} = c$. As this adjustment is varied by continually adding to one wick and diminishing from the other, the joint-flame is continually increased, until the whole of one wick is superimposed on the other. This process may continually be repeated with the other elements of the integral, until the whole illumination is reduced to one long wick, affording φλογὸς μέγαν πώγωνα; the greatest quantity of pleasure is obtained out of any given quantity of material, by assigning the whole of that given quantity to *one* individual. So, if any of the functions of the previous heading, α, β, γ, &c., had the second differential continually positive, the abnormal functions would eat up ~~the normal ones, and~~ each other. As this deduction is contrary to common experience, it follows that $\frac{d_2 \pi}{dy^2}$ must be negative for the higher values of y at least. And this conclusion agrees with Wundt's pleasure-curve, of which, as already mentioned, the higher part is concave, the lower part is convex, to the abscissa. Thus, then, when the limits of the integral are variable, $\frac{d_2 \pi}{dy^2}$ may be treated as negative by the Hedonistic Calculus.

Again, it may be expected that the investigation of the

[1] "Methods," ii. ch. 4, § 3.

second term may often be dispensed with in virtue of Mr. Todhunter's principle, that where by the data y can only vary from a given curve in such a manner that the first term of the variation must be negative, there the given curve affords a maximum. It may be hoped that more felicitous applications of this principle than that indicated by way of illustration under the previous heading, will be forthcoming. For practical utilitarianism must generally aim, not at the best conceivable, but the best attainable; must seek, as in Mr. Todhunter's ingenious problem[1], not the shortest course abstractedly, but the shortest course given rocks ahead.

Lastly, it seems agreeable to the procedure of the Calculus of Variations, at least to its (sufficiently secure) procedure before the discovery of criteria for the second term, to assume *à priori* in some cases that there must be a maximum. And this no doubt precarious assumption is perhaps sufficient for the working of the second problem. But it is not sufficient for the argument of the first problem, whereof, it may be remembered, the essence is

$$\left(\frac{dF}{dy}\right) = \text{constant}, \left(\frac{d_2F}{dy\,dx}\right) = +, \left(\frac{d_2F}{dy}\right) = -:$$
$$\therefore \frac{dy}{dx} = +.$$

(III.) The evidence of the third and fourth conditions is much the same. First, it may be observed, that in order to deduce *some* conclusion, the particular assumption made in the postulates is not necessary. It is necessary, not that $\frac{d\pi}{dy}$ and π should continually increase with the progress of evolution, but that they should increase with *some* ascertainable property[2]. The selected property might e.g. be the speculative powers (which have been thought the criterion of progress), or "civilization" in the sense attached to it by Mill[3]; or "progress" or "cultivation" in any definite

[1] "Researches," &c.

[2] The x in the formulæ may correspond to the selected property.

[3] "Dissertation and Discussion."

sense ; or, for a utilitarian of the Paley type, the prevalence of the Christian religion. It is not even necessary that the increase of π and $\frac{d\pi}{dy}$ should be known to be dependent on any *one* factor. Supposing π and $\frac{d\pi}{dy}$ to increase together[1], and loosely designating their increase as the increase of "felicific power," capacity to extract pleasure from given material means, we have the following substantial proposition. Unto him that hath (much felicific power) shall be added (the means of happiness) ; and from him that hath not (would not have had much felicific power) shall be taken away (by not calling him into existence) even the little that he hath (even the little happiness he could have enjoyed, the little means to which, if existent, he would have been entitled according to the unequal law of distribution). Felicific power, though not evidenced by any *one* set of marks, is still, wherever found, to be encouraged and privileged. Or, to adapt the illustration already employed (p. 52), the peoples would still pursue "the upward road," though the elevation did not consist of *one* heaven-climbing ridge, though the loftiest mountains lay apart in diverse regions.

The evidence of the third and fourth conditions, in the more limited sense in which they are here postulated, may now be intimated. Or perhaps it is unnecessary to attempt to demonstrate the increase of felicific power with evolution ; since this whole argument is chiefly addressed to the (inexact) utilitarian ; and the utilitarian is apt to concede what is to be demonstrated, as Mr. Sully has shewn, and as was noticed on a previous page[2]. It may suffice, then, to add to what was there said the following few trite remarks.—Even as to the pleasures of sensation, or, at any rate, emotions not much removed from sensation, the cultivated man has an advantage in the variety of his tastes. He is like the cultivated land already de-

[1] Even if these properties did not hold together, there would still be a fragment of a chain.

[2] Supra, p. 31.

scribed (p. 52): if not the very most fertile land, at any rate a very large tract of averagely fertile land. As we ascend from the region of sensation, we may seem to come first to what Fechner[1] calls relations between sensations. For in the arithmetic of hedonism, the whole is not to be equalled to the sum of the parts, but to the sum + the relations of the parts. But the more "relational" nature, as Herbert Spencer would say, is that which is more highly evolved. Next may be placed the intellectual powers generally, the connexion of which, both with felicific power and with evolution, it is unnecessary to illustrate. As another advantage on the side of evolution may be distinguished (so far as distinction between properties so closely clustered is possible), what Herbert Spencer has described as quantity of feeling[2]:—

"With a qualification to be hereafter made[3], the higher the evolution rises the stronger do the emotions become. For as the increasingly-complex emotions successively developed result from integration of pre-existing groups of actual and nascent sensations, the resulting totals must grow continually larger[4]," &c.

With greater quantity may, perhaps, be enumerated more pleasurable quality[5], more sympathy, æsthetic feeling, sense of dignity. If we fix our attention upon social pleasures in particular, there also the advantage may seem to be entirely on the side of progress. Nor is there, perhaps, any very important exception to this statement, but the fact which Adam Smith and many others have noticed, that advancing civilization, affording increased means and inducements to migration, tends to break up the ties of consanguinity, and generally the fixedness of social relations. The loss incurred may seem in the last analysis to be the impoverishment of that "ideal emotion[6]" which, as Mr. Bain has so felicitously shewn, is

[1] p. 161.
[2] "Psychology," § 215; "Mind," No. I.
[3] Cf. infra, p. 73.
[4] "Psychology," p. 486.
[5] It need hardly be said that "quality" is not here employed to denote ncommensurability. See supra, p. 26.
[6] "Emotions and Will," third edition, ch. v. § 11.

fostered by fixedness of relations. It is to be trusted that this impoverishment will be compensated by an exceeding weight of gained felicity. For instance, for the separation caused by increased means of locomotion, some reparation is made by still further increased means of communication.

It really seems sufficient to consider for a moment how inferior in all these respects is Mr. Pongo to almost any of his visitors. Not that it is necessary to prove, that every more highly-evolved sentient possesses *all* these advantages in a higher degree, but that he *tends* so strongly to possess all of them, that he is almost certain to possess higher felicific power in virtue of some of them. It may be objected, indeed, by the Pessimist that, if the beast has "lower pleasures," he has also "lower pains." But this slavish whisper in the triumph of evolution has for ever been silenced by Mr. Sully, who in his refutation of pessimism, and in an earlier essay[1], has abundantly and lucidly, though not quite unreservedly, deduced the proposition here postulated. But there is one advantage not so clearly all on the side of evolution, as to which neither the example of the ape, nor the theory of the philosopher, will help us. How do we know that Pongo has not a superhuman pleasure in simple sensations and muscular movements? How do we know, as Mr. Sully asks, that birds, attracted by bright colours, do not feel a simple pleasure more intense than we can realize? Can we be certain that π does not lose in respect of β what it gains in respect of k? It may be hoped that advancing psychophysic will throw some light on this question. In the meantime, as far as regards the most momentous practical bearing of the theory, the alternative presented by Mill between a limited cultivated, and an unlimited, joyless, ant-like society, it does not seem that the question just raised is of much practical importance. Indeed, it might be contended that even as to the simpler pleasures of sensation and of movement, the cultivated man, in virtue of his better education, of μουσικὴ and

[1] Essay xiii. Sensation and Intuition.

γυμναστικὴ, would possess an advantage, not so much over the savage, as over the pauper.

(IV.) It remains to add under this heading an important modification of the preceding theory. Up to this, sentients being regarded as so many lamps of different lighting power, the questions have been what lamps shall be lit, and how much material shall be supplied to each lamp, in order to produce the greatest quantity of light. And the answers, neither unexpected, nor yet distinctly foreseen by common sense, are, that a limited number of the best burners are to be lit, and that most material is to be given to the best lamp. But the conception more appropriate to the real phenomena is, that a large portion of the material to be distributed is applied not to be burned by the lamp, but to construct and repair it. Here, then, is a further modification of both problems.

As to the first problem, it is to be regarded as hitherto applied to those cases only in which the distribuend is sufficient to supply all the distributees with necessaries, and to spare. For it may be doubted whether the condition $\frac{d_2\pi}{dx\,dy} = +$, which is essential to the first problem, would otherwise hold good, at least where a different amount of necessaries may be required by different individuals. Where such a difference does not exist, the condition probably holds good, and the reasoning proceeds as before, except that π is to be regarded as negative, as denoting *pain*, when y sinks below that amount of necessaries "queis doleat natura negatis." As before, a law of privilege is deduced. But it is probable that the inequality of privilege is in this case less. For this inequality is expressed by the inclination of the tangent of the curve y, by $\frac{dy}{dx}$. And $\frac{dy}{dx} = -\frac{\left(\frac{d_2\pi}{dy\,dx}\right)}{\left(\frac{d_2\pi}{dy_2}\right)}$. And, while there is no reason to suppose that the numerator of this fraction is greater when y is small, there is reason to suppose that the denominator is greater. For this suppo-

sition corresponds to the fact that *minus* pain is sweeter than *plus* pleasure, that what is given to the most miserable is given most felicifically: a fact upon which Butler has insisted with his usual force in his Sermons on Compassion, and which is abundantly confirmed by common experience; even the optimistic Adam Smith admitting that pain is more pungent than pleasure. Wundt also expresses well the same fact in his diagram[1], where the curve of pain descends to ever lower depths, while the curve of pleasure soon attains her "wendepunkt[1]." It seems, then, that if there is not enough for all, no one should get very much. Perhaps all will have to forego.

Here it may be expected that the very rich will go away sorrowful from utilitarian teaching. For the conclusion just reached appears pertinent to the actual conditions of wealth and population. For it may be doubted whether a consistent utilitarian, a man like the hero of Aurora Leigh, who was capable of acting out a great principle, could, in view of the stupendous mass of misery around him, allow to himself and to his family any of the refined pleasures and luxuries of civilization, or anything but the barest necessities of a painless and a useful life,—but for the reflection that, by any wholesale beneficence of the kind contemplated, the consequence in the second generation would be to have diminished, not the poverty of the poor, but the cultivation of the cultivated, not the quantity of misery, but the quantity of happiness, on the earth. That reflection is one of the most important of the limitations which utilitarianism imposes on itself; (those limitations[2], in virtue of which the utilitarian proprietor, though to the last he holds his property as tenant-at-will only of the general good, is yet undisturbed in that tenancy). Behind the first problem, to distribute over

[1] "Physiolog. Psychol.," p. 432. To illustrate the present reasoning, one should suppose an infinite curve of pain below the threshold, as well as beyond the "wendepunkt."

[2] As to the limitations which both species of hedonism impose upon themselves, see the "Methods."

the present generation, looms the second problem, to select from posterity[1].

With regard to the second problem, prior to the consideration lately introduced, concerning "necessaries of life," it might formally have been argued (p. 48), that out of the same quantity of material a hundred philosophers would elicit more happiness than a hundred capuchin monkeys. But perhaps the material which would keep a hundred little monkeys in health and happiness would not feed twenty philosophers! There is then a "threshold"—preliminary to the Fechnerian—consisting of the necessities of life, all that must be pre-supposed before the sentient begins to experience pleasure ("satur est quum clamat Horatius 'Euœ'"). To the necessities of the individual must be added his contribution to the necessities of the social state to which he belongs. The "threshold" thus given (say T) must be deducted from the individual's share (y) before the quasi-Fechnerian theory can be applied. Now, it is a nice question whether T is to be regarded as increasing or decreasing with the progress of evolution (civilization, &c.). On the one hand, may be urged the costliness of vulgar pleasures,

[1] Not only from the second generation, but from "the innumerable multitude of living beings present and to come." This extension of view is not always favourable to privilege. For example, *primâ facie*, unequal legislation directed against the influx of Chinese labour might be justified, on the supposition that, if on a large scale Chinese competed successfully with Aryans, an inferior race would inherit the earth. But this *primâ facie* correspondence between exact utilitarianism and commercial selfishness would disappear, *if* it were probable that the inferior race, not retarded by unequal laws, would catch up the superior in the race of evolution, and become ultimately as highly civilized,

"as completely so,
As who began a thousand years ago."

The difference of civilization during a short interval of such pursuit might be neglected, or rather, would be counterbalanced by the invidiousness and deteriorating tendencies of unequal legislation.

Of course, it will be understood that examples are put forward in these pages τύπῳ and παχυλῶς, and without practical qualifications. It has been sought, not to clothe generality in circumstances, but to exhibit boldly the conception of exact utilitarianism—with a not unfelt sacrifice of delicacy to clearness.

the simplicity of refinement, the frugality of the philosopher. On the other hand, there appears much weight in what Strauss says[1], arguing against communism, about the costliness of art and cultivation. And as to the philosopher, even Aristotle admits, δεήσει δὲ καὶ τῆς ἐκτὸς εὐημερίας. And the "self-sufficiency" which Aristotle attributes to the philosopher, theorizing independently of apparatus, is much less true of the less metaphysical modern savant. And even for the philosopher of Aristotle, there seems to be postulated a πολιτεία abounding in ἠθικὴ ἀρέτη; and ἠθικὴ ἀρετὴ postulates an abundancy of external means. Upon the whole, when we consider the *preliminaries* of art and science, and of moral virtue —whether regarded as ancillary to intellectual, or with a more English feeling, and *not* δευτέρως;—when we consider the material conditions of moral perfection, of the beauty of virtue, and the not in every sense "*unbought* grace of life;" then what was previously assumed[2] must be very cautiously assumed, that, wealth and population being stationary, there is possible, through a whole society, an appreciable, a continual advance in evolution, cultivation, civilization, or whatever is the mark of felicific progress. It rather seems as if wealth, and number, and quality are so related, that at a certain point it would be impossible to heighten the third, without either increasing the first, or diminishing the second; though, doubtless, society may be far from thus being saturated with virtue.

Perhaps it might be safe to say, that the increase of the threshold is just balanced by the increase of the distribuend (cp. p. 53), due to increasing civilization? At any rate, some opinion must be formed about the "threshold," before we enter upon "the upward road." To take, by way of illustration, a simple and abstract, yet momentous, case, the case which Mill may be supposed to have in view when he writes in favour of the stationary state. Let $\Sigma \pi = n F\left(x, \frac{\phi(n x)}{n} - T\right)$; where n is the num-

[1] "Der alte und der neue Glaube," iv. 80.

[2] Supra, p. 48.

ber of population, x the order of evolution, T the threshold, $\phi(nx)$ the distribuend depending on the number and quality (supra, p. 53), where the partial differential $\left(\frac{dF}{dx}\right)$ is always $+$. As long as T does not increase with x, an indefinite progress in evolution is desirable, *τῆς ἄνω ὁδου ἀεὶ ἐξόμεθα*. But if the threshold increased with evolution, then we should tend to a "stationary state," not only wealth and number, but also, what Mill hardly contemplated, cultivation, evolution, stationary. This state might be symbolized by the equations $\frac{d}{dx}\Sigma\pi = 0$, $\frac{d}{dn}\Sigma\pi = 0$.

To sum up the indications of exact utilitarianism. With regard to the theory of distribution, there is no indication that, at any rate between classes so nearly in the same order of evolution as the modern Aryan races, a law of distribution other than equality is to be wished. The more highly evolved class is to be privileged when there is a great interval, as there is between man and ape, as there may have been between the ranks and races of the ancient world.—With regard to the theory of population, there should be a limit to the number. As to the quality, it were to be wished that the quality should be as high (in the scale of evolution) as possible, *cæteris paribus*, and as long as the number is not impaired. But if number and quality should ultimately come into competition, as seems to be not impossible, then the indefinite improvement of quality is no longer to be wished. There is here no paradox that the means of subsistence shall devolve upon the most cultivated, however few, (comparable to the paradox entertained by Aristotle, that political power should devolve upon the most virtuous, however few). Not the most cultivated coterie, not the most numerous proletariate, but a happy middle class shall inherit the earth.—It is submitted that these conclusions are acceptable to common utilitarianism, and, if not to common, at least to good, sense.

These observations upon so vast and dear concerns might well appear jejune, and almost impious, if they purported to solve, or even accurately state, the stupendous problems of utilitarian philosophy. But they suffice perhaps to extricate a clear, and, as it may be termed, a mathematical conception of exact utilitarianism: to elucidate the implicitness of that doctrine of the "Methods" in the less precise (though cultivated) forms of the science of the end. The Method is strictly introspective; whether egoistical, as shewn to be conceivable (above, pp. 28—30); or more faithfully following philosophical intuitionism, which it at any rate resembles, in the essential process from the "unconsciously implicit" to the philosophically evolved. Where the great body of moral science is already gone before, from all sides ascending, under a master's guidance, towards one serene commanding height, thither aspires this argument, a straggler coming up, *non passibus æquis*, and by a devious route. A devious route, and verging to the untrodden method which was fancifully delineated in the previous section; so far at least as the mathematical handling of pleasures is divined to be conducive to a genuinely physical ethic, *προοίμια αὐτοῦ τοῦ νόμου.*

NOTE A, to p. 7.

THE admirer of Locke—if he were as concerned to defend, as others to attack, unguarded positions which English philosophy in its onward march has long since left behind—might suggest an interpretation of Locke unfavourable to one of the criticisms in Mr. Green's Introduction to Hume's Treatise, Part II. Mr. Green observes, referring to Locke, II. 21, §§. 61, 63, 67, "By these wrong judgments, it will be observed, Locke does not mean mistakes in discovering the proper means to a desired end (Aristotle's *ἀγνοία ἡ καθ' ἕκαστα*), which it is agreed (?) are not a ground for blame or punishment, but wrong desires—desires for certain pleasures as being the greater, which are not really the greater[1]." And he proceeds to argue that since, according to Locke, every "pleasure is just so great, and no greater, than it is felt," "is really just as it appears;" and since, in fact, "in whatever sense it is true of the 'present pleasure and pain,' that it is really just as it appears, it is equally true of the future[2];" it is not open to Locke to speak of wrong judgments in comparing present pleasure and pain with future.

Now Locke speaks of (*a*) "the wrong judgment we make of present and future pain when they are compared together[3]," of the wrong judgment which lies "in comparing the greatness of future good and evil[4]." But then he distinguishes (*b*) "another sort of wrong judgment, which is concerning good and evil, as it is considered to be the cause and procurement of pleasure and pain that will follow from it[5]," "things good or bad in their consequences," &c. (§ 66). Compare in § 56, "A man may suspend the act of his choice from being determined for or against the thing proposed, till he has examined whether it be really of a nature *in itself and consequences* to make him happy or no." The two cases appear to be distinguished by the numerals prefixed in the earlier editions to § 63 and § 66.

Now it may be questioned whether Mr. Green has seized the

[1] Introd., § 12. [2] Ibid., § 13. [3] End of § 55, Locke, ii. 21.
[4] End of § 63. [5] Ibid.

definition of these classes, which will be here suggested with brevity and hesitation. In both cases there is, owing to inattention, a mistake as to a matter of fact, the pleasures and pains connected with a phenomenon, the "thing proposed;" in the latter case (*b*) a mistake as to remote consequences; in the former case (*a*) as to co-existences and immediate consequences, (especially such as are apt to be associated with the phenomenon, and, as Mr. Lewes might say, combine with it so as to constitute an integral concept [1]). Thus, to take Locke's principal example, a mistake of the class (*a*) is, not considering that "the manna in heaven will suit every one's palate." A mistake of the class (*b*) would be, not considering that the breach of a particular precept might entail forfeiture of heaven. The mistake and the regret of the agent are doubtless in both cases much the same; in both

ἦ λάθετ', ἦ οὐκ ἐνόησεν ἀάσατο δεμεγὰ θύμῳ.

This construction being allowed, the answer to Mr. Green's objection above stated may be metaphorically indicated. Locke says that the brightness of objects is only to be ascertained by looking at them; but that as to the brightness of distant objects we are liable to be mistaken. Mr. Green accuses him of inconsistency, reminding us of the optical fact that objects (tend to) appear equally bright at all distances. It is here suggested that, according to Locke, the mistake in the case of a distant object is not as to the contemplated parts of the object, but as to the parts on which the eye, solicited by nearer objects, is not allowed to dwell.

The interpretation here suggested may be reconciled to, and may even reconcile, some passages in Locke which, at first sight, might seem to conflict, not only with the interpretation, but with each other [2]. It agrees also sufficiently well with the comments of Leibnitz [3]. He seems to admit [4] that there is not much difference between wrong judgment "in comparing the greatness of future good and evil" (our *a*), and wrong judgment as to future consequences of the present (our *b*); in so far, at least, as the latter judgment is due to negligence. He says [5] that when (our *b*) we "judge so much evil does not really depend on them as in truth does," (I quote Locke's words), "on doit tomber dans

[1] See Hist. of Phil., vol. ii. ep. 9, § 3. [2] Compare § 55 with § 59; and again, § 35 with end of § 45, and latter part of § 53. [3] *Nouveaux Essais*, &c. [4] Ad Locke, ii. 21, § 63, sub finem. [5] Ad § 66.

l'espèce precédente de faux jugement ou le bien ou mal à tenir est mal representé" (our *a*).

The point is not very important, perhaps not quite visible; the chapter of Locke is written, as Mr. Green admits, *currente calamo*, and affords too much opportunity for those who, as Locke would say, like to "stick in the incidents:" so that any explanation of its endless subdivisions must be advanced with infinite diffidence, especially if it is not very emphatically confirmed by Leibnitz, and is ignored by Mr. Green. Only it seems rather slippery ground for the rolling of the cannon which are to silence English philosophy, and establish the empire of Hegel.

NOTE B, to p. 9.

To avoid misconstruction, at the risk of repetition and truism, I append the definitions of "Interested" (selfish, &c.); in answer to the question, "Are all actions interested?"

I. "Interested" may mean, prompted entirely by self-regarding unsympathetic impulses, or, (the same *plus* a connotation of excess, a note of blame) selfish, φιλαυτός, in the popular sense distinguished by Aristotle, namely, πλεονεζία, of money, bodily pleasures, &c. In this sense, surely, all actions are not interested. The hypothesis of Mandeville starts, indeed, from a *vera causa*, but it fails in the ratiocination. It will not account for all the phenomena—for an Antoninus, or a Howard. It will account for many of the phenomena; so would a converse hypothesis, (as Brown shews), that all actions spring from a tender regard to the welfare of others. (Hutcheson accounts for the attraction of the gladiatorial shows as a case of compassion!)

II. "Interested" may mean, prompted by affections, heteropathically compounded of the preceding class. Much of the antipathy which is felt towards this doctrine, is removed by the observation that the composition in question is *heteropathic*, the compound unlike the components, that by the magic of mental chemistry we may "begin with selfishness, and end in self-devotion." To the doctrine as held by Hartley and most of his followers, the early appearance of our benevolent feelings, and their ineradicability, have been objected. But the theory of evolution —that the component associations are ancestral—is free from such objections, and is confirmed by a large induction.

III. "Interested" may mean, prompted by desire, desire in a limited and technical sense, such as that defined by the Mills,

the *idea of a pleasure*, (a possible future pleasure, the prospect of which tends to excite action). The case of habit is allowed by J. S. Mill, and would be generally allowed, to be an exception. The following are more dubious cases of non-hedonistic action; the affection of Butler, the reason-originated moral action of Clarke or Kant, the fixed idea of Bain. Perhaps the point in question hardly reaches the minimum visibile of consciousness. The blurred and almost invisible type of reflection is only to be deciphered by the aid of a parallel inscription in the clear and firm characters of Physical Science.

NOTE C, to pp. 10, 27.

It were to be wished that Mr. Green had enhanced the great debt already due to him from the non-Hegelian public, by exhibiting in a greater variety of lights what he regards as the "central question of Ethics," the "constitution of the desired object[1]."

He considers that Hume allows no place to reason, (§ 1) in the "constitution of objects that determine desire," or only so much (§ 48) "as renders selfishness possible:" where the last words, as appears from the context, allude to the function of reason in discovering means to ends. But it should be remembered, and very possibly has been mentioned by Mr. Green, that Hume allows another function to reason in reference to desire; namely, discovering a relation which is a cause, or prerequisite, of sentiment. The proportions of a column must be discerned before its beauty can be felt. In the Essays this view is stated by Hume with his usual clearness, and maintained with his usual consistency[2]. And it deserves consideration, whether any one (as Mr. Sidgwick would say) is "competent or concerned," competent by attaching distinct consciousness to his verbal distinctions, or concerned for the sake of morality, to dispute Hume's view in favour of any other theory of the power of reason to originate action.—For example, the criticisms of Reid upon Hume might almost be accepted by a Humian.

There is another distinction in reference to desire, the benefit of which is denied by Mr. Green to Hume. This is the distinction made by Butler between "desire of pleasure" and "affection;" in the language of Mr. Sidgwick, between hedonistic

[1] Introduction to the Moral Part of Hume's "Treatise on Human Nature."

[2] On the Standard of Taste. Inquiry concerning Principles of Morals, sect. i. Ibid., Appendix I. sect. iii.

and non-hedonistic desire. In the Essays, Hume has indicated this distinction in a well-known passage at the beginning of the "Dissertation on the Passions;" and has maintained it in the Appendix on "Self-love[1]." In what Mr. Green regards (Introduction) as an important note (omitted by Hume on maturer thoughts), there seems to be an allusion to this distinction; in which case, the remarks in the Introduction may require modification. It is a nice question for the otiose student of the History of Philosophy, whether Hume observed and preserved the distinction in question, in his juvenile "Treatise on Human Nature." But even if he did not do so, which Mr. Sidgwick seems to doubt (Review in the "Academy," 1875), and if he erred in not doing so, since he did so in his mature and considered works, it is open to question the propriety of associating the name of Hume with his youthful errors. It is possible to disregard the Treatise, and to regard the Essays as the characteristic and influential version of Hume's philosophy.—The flower with the fertile germ of English philosophy: the master-piece and model of a mode of thinking which is peculiarly English, that something of practical and palpable and even in speculation tangible; so strikingly, whether favourably or not, contrasted with the style of the school which is advocated in the Introduction. (One could not perhaps more briefly indicate a difference, without expressing a preference, than by the little fact that a genius of the calibre of Macaulay could appreciate the philosophy of Hume, could sigh, as Gibbon sighed, for the fame of Hume, while he turned from the perusal of the Kantian pages without having received a single idea[2].) If such were the importance of the Essays, the blows which are aimed at the Treatise, even when well aimed, would fall upon the scaffolding and not the edifice of English philosophy. It would be as if one brought up against some Liberal leader the extravagancies of his youthful ardour for liberty, the hyperboles of his debates at college, in order to discredit the statesman of mature years and the Liberal party itself. It would be like ransacking the studio of some great artist for his crude designs and boyish sketches, by the exposure of which to damn both the master and the school. The analogy and defence presume, indeed, a certain view of the historical position of the Essays; concerning which a different view is taken in the Introduction[3]. A difference on such

[1] And throughout the Inquiry; Conclusion, Part II., &c.

[2] Life of Macaulay, p. 243.

[3] Cf. Mr. Green's Preface.

a subject from the writer of the Introduction must be advanced with the utmost diffidence: but in justification of such a difference may be cited the well-known views of Hume himself, who certainly would associate his name, and fame, and philosophical influence with his mature and considered Essays. The views and emphatic words of the great author himself, though quite inappropriate to the perfect candour and elevating moral tone of the critic, are not without bearing on the validity of the criticism as aimed against the author's school. "He was sensible of his error in going to the press too early, and he cast the whole anew in the following pieces (the Essays, &c.); where some negligences in his former reasoning, and more in the expression, are, he hopes, corrected. Yet several writers, who have honoured the author's philosophy with answers, have taken care to direct all their batteries against that juvenile work, which the author never acknowledged, and have affected to triumph in any advantages which they imagined they had obtained over it. A practice very contrary to all rules of candour and fair-dealing, and a strong instance of those polemical artifices which a bigoted zeal thinks itself authorized to employ. Henceforth the author desires that the following pieces may alone be regarded as containing his philosophical sentiments and principles[1]."

But, to continue the subject of "desire;" it appears that from the Hegelian point of view, Locke excludes "every motive to action but appetite proper, and such desire as is determined by the imagination of animal pleasure or pain[2]." Locke certainly did not seem to himself, nor does he seem to his admirers, inconsistent in writing: "Among the simple ideas which we receive both from sensation and reflection, pain and pleasure are two very considerable ones. For as in the body there is sensation barely in itself, or accompanied with pain or pleasure; so the thought or perception of the mind is simply so, or else accompanied also with pleasure or pain, delight or trouble, call it how you please[3]." "By pleasure and pain, I must all along be understood (as I have above intimated) to mean not only bodily pain and pleasure, but whatsoever delight or uneasiness is felt by us, whether arising from any grateful or unacceptable sensation or reflection[4]." "Delight or uneasiness, one or other of them, join themselves to almost all our ideas, both of sensation and reflec-

[1] Hume's Advertisement to the Essays.
[2] Introduction, § 5.
[3] Essay ii. c. 20, § 1.
[4] Ibid., § 10.

tion: and there is scarce any affection of our senses from without, any retired thought of our mind within, which is not able to produce in us pleasure or pain[1]." "Pleasure and pain, there being pleasure and pain of the mind, as well as the body[2]," &c.

Nor do the motives open to Hume appear more elevated from the Hegelian point of view[3]. The lover of Hume, the Hume of the Inquiries, contemplates with amazement the attempt to reduce, upon the principles of Hume, all virtuous motive to the "Pride" of "Respectability[4]." Surely of the fairer half at least of virtue, of the "qualities" agreeable and useful to others, the prime motives are the passion of benevolence, the pleasures of sympathy, which the genial philosopher has painted with a fond repetition, in every aspect of their charms, in all the colours of his art.

NOTE D, to p. 16.

Can following nature be taken as the standard of morality?

I. "Following nature" may mean, following each impulse as it rises, a meaning which is noticed by Butler only to be excluded. It is indeed the negation of all rule and standard.

II. "Following nature" may mean—not an anarchy, but a despotism—following the impulses which are *de facto* strongest. Butler excludes this meaning also. Yet it deserves to be considered, whether a solid and even a beautiful morality cannot be founded on this position.—In fact, What else is the foundation of the systems of Hume and Mill? Hume, when he considers our "interested obligation" to practise those virtues which he has shewn to be cases of beneficence, can only paint the charms of benevolence, virtually appealing to it as the strongest desire. Mill postulates that all action is a product of desire. What other inducement to utilitarian philanthropy can he consistently offer, but mass of sentiment, actual preferredness, strongest desire[5]?

III. "Following nature" may mean, carrying out in our conduct the Divine design. It is on this ground that Butler sometimes establishes the supremacy of conscience as natural: at other times, no doubt, he executes the briefer circle exhibited by Mr. Sidgwick: the language of the Stoics hovered nearer to the latter position. If now it is asked, "Why is the Divine design to be realized, the Divine will to be obeyed;" Butler replies, "Because it is our *duty* to God[6]." His circular course of reasoning

[1] Essay ii. c. 7, § 2. [2] Ibid., 21, § 41. [3] Introduction, § 48, &c. [4] Ibid., sub finem. [5] Cf. supra. [6] Sermon iii. § 44, ii. § 40, Whewell's edition.

meanders level with its fount. It is fair to add, (1) that Butler is ready to strengthen his argument by the Divine sanctions; (2) that his argument may be an *argumentum ad hominem*, to the man (if such there be) who may have a stronger sense of duty to God than to his fellow-men. But, indeed, the whole argument from final causes, whether applied, as in the first three sermons, to the general scope of human nature, or, as in the later sermons, to the functions of the particular parts, *nihil parit.* For how do we know *what* is the design of God, or that He designed it to be the model for our conduct? Is not Butler, as he himself would say, forming a world upon hypothesis, making very free in speculation with the attributes of the Deity? He takes for granted, that the original in human nature is fresh from the hand of God, an inviolable model for man. But can this be maintained if the original is proved to be often pernicious? Such a proof involves in a common fall, a *fourth* doctrine, which seems to stand somewhat apart from Butler's characteristic position.

IV. "Following nature" may mean, taking the inartificial as the model of our conduct, (not so much from piety, as) from the assumption that the inartificial is beneficial. Here may be placed Rousseau's declamations in favour of a return to the state of nature, perhaps the declamations of Latin authors in favour of a return to a Sabine, or a Saturnian age, and (according to Mr. Mill at least) much of current literature. But the fundamental postulate, the goodness of nature, is amply refuted by Mr. Mill. The supposed *alma mater* is exhibited as a very *sæva noverca.* Nature in general presents neither justice nor mercy. Human nature in particular has original affections, altogether evil and to be suppressed. The only question is, whether the persons so affected should not also be suppressed? —the question lately asked about the Turks.

V. The remarks of Mill appear to strike, but with diminished force, another band of followers of Nature, whose watchword might be generally described as "forwarding evolution," (not by way of axioma medium of utilitarianism, as in the preceding pages[1], but as an end in itself). One of the ablest expositors of such a system, Professor Clifford, writes[2], "Our question about the best conscience will resolve itself into a question about the purpose, or function, of conscience," namely, the "unconscious purpose" of Nature. But *why* is this purpose best and to be

[1] Section 2, V. and VIII. 3.

[2] Article on Right and Wrong in the "Fortnightly Review," 1875.

adopted? The answer must be some such proposition as "The purpose of Nature is the end of action, the standard of morality;" a proposition abundantly refuted by Mill's Essay on Nature. This first proposition is made applicable by another assumption, viz. that the purpose for which conscience exists, its "function," is the preservation of society in the struggle for existence. Now this second proposition ignores the fact that, even on Darwinian principles, there is more than one "unconscious purpose" in Nature, there is a purpose to beautify, as well as to preserve. In the words of the Darwinite Hækel, "By far the most important of these different forms of sexual selection in man, is that form which is the most exalted, namely, psychical selection, in which the mental excellencies of one sex influence and determine the choice of the other. The most highly intellectually-developed types of men through generations, when choosing a partner, have been guided by excellencies of soul," &c. Suppose now this second purpose, and the ideal which it supplies, to conflict with the first (as may well be in the reasons for, and the feelings against, the rights claimed for women), must there not be an appeal to a superior principle, as arbiter, to utilitarianism?

VI. It is hardly worth while to distinguish as a separate form "taking account of natural laws," e.g. of gravitation in walking over a bridge[1], except as to certain laws of mind, e.g. Mr. Barratt's argument, all action is hedonistic. The argument (3) on p. 13 *supra* is of this "nature."

VII. The "Natural Law" of Miss Edith Simcox appears to involve both this and the previous signification. But it is impossible upon short notice, in a brief note, to treat adequately a work so recent and so profound.

NOTE E, to p. 16.

As Professor Fowler says, the student of Mill's "Empirical Laws" will find more difficulties raised than solved. Here are a few; none, I think, fatal to the view in the text.

The definition of empirical law halts between that given at the beginning and at the end of § 1, ch. 16, Book III.—an obscurity well avoided by Professor Fowler. If the latter definition be adopted, it is difficult to see why a law, of which the proof rests only on the method of agreement, should be empirical, pending the application of the method of difference[2]. The method of

[1] Mill, on Nature.

[2] Ch. xvi. § 5.

difference might confirm, without deriving, the law. Much less can the law of causation[1] be an empirical law in the latter sense. It is the one law which does not admit of derivation. If, as seems evident, the first statement[2] be the definition, the second[3] only the description, there is still some difficulty in considering the law of causation as an empirical law—a difficulty, however, fully cleared up by Mill.

Among the "signs by which we may judge that, even if an observed uniformity be a law of causation, it is not an ultimate, but a derivative law," are enumerated *two* of the cases of derivation described in a previous chapter. Why not also the *third* (subsumption)?

The limits within which empirical laws can be received as true are, according to chapter xvi. § 4, those of time, place, and circumstance; but in chapter xix. only the two former are dwelt upon.

Ought not we to speak more provisionally than Mill is wont to do of *ultimate* laws? How do we know that the chain of causation connecting two phenomena is not divisible into an infinite number of links? or that it is not infinitely broad?—a sort of difficulty well pushed by Mr. Shute, and of which a practical solution is perhaps to be found in Hume's "Theory of Belief." But this ἄλλης ἂν εἴη ζητήσεως. The terms, "cause," "conditions," &c., are used in the text in the rough sense, which is found sufficient for the practical purposes of inductive logic.

NOTE F, to Sect. 2, V.

Mr. Green would make short work of hedonism.

The objections on § 7[4] may be treated under two heads.

(*a*.) It may be objected that it is not open to the nominalist to employ such terms as "pleasure in general[5]." (1.) Now it is not clear why this objection is introduced *à propos* of Locke. To Locke, at least, it was competent to speak of an abstract idea of pleasure in general, and therefore also of "pleasures of a sort we are unacquainted with[6]"—ideas formed by combining the idea of pleasure "separate from circumstances of real existence" with imaginary circumstances. (2.) But indeed it deserves consideration, whether it is not open to the nominalist also to

[1] Ch. xxi. § 3. [2] Ch. xvi. § 1. [3] Ibid.
[4] Introduction, § 7. [5] Cf. § 47. [6] § 14, cf. § 50.

employ the term "pleasure in general" agreeably to the doctrine of Berkeley and J. S. Mill, just as much as any other general term. The nominalist may speak of a new kind of pleasure, just as well as of a new fashion of shoes. He may consistently be a lover of pleasure-in-general, just as he may be a dog-fancier, without, on the one hand, limiting his attentions to a particular species of cur, nor, on the other hand, separating from particulars the entity which Mr. Ruskin would call "dogity."

(*b.*) What further objections are contained in the passage under consideration (§ 7), underlying such phrases as "vain attempt to get a definite by addition of indefinites," may appear to the hedonist as ingenious as the Eleatic arguments against motion, as appropriate in morals as those arguments in strategics. For the hedonist the difficulty *solvitur* by increasing as much as he can the pleasures of others, or his own. Mr. Green says the hedonistic end "has no more meaning than 'the greatest possible quantity of time' would have." The hedonist accepts the parallel, and, while he admits with Berkeley that there are difficulties about pure time, which would "gravel even a philosopher," makes his daily arrangements so as to have the greatest quantity of time at his disposal.

Mr. Green's Note in Mind (No. VI.) on "Hedonism and ultimate Good" may appear to the hedonist equally impalpable. A man, "if he experiences a pleasure every hour for the next fifty years, will have no more in possession, and will be in no better state, than if he is pleased the next moment, and then comes to an end," &c. The hedonistic motive is "idea of pleasure in the future," not in the *paulo post futurum.*

NOTE G, to Sect. 2. V.

If we accept Mr. Sidgwick's conclusion (even subject to the slight and hardly practical modification suggested above, p. 28), that egoism, *quâ* egoism, cannot consent to a clear and uncompensated self-sacrifice, what becomes of the position of the "Fixed Idea" sympathy, in the composite system of Mr. Bain's utilitarianism? This system is grounded on (egoistic) normal volition—the motives supplied by rewards and punishments (especially the latter). It culminates in sympathy, a fixed idea leading the agent to a clear self-sacrifice of personal pleasure. (But for this addition, utilitarianism under Mr. Bain's treatment would be, as Professor Blackie hastily says it is, "reduced to the naked-

ness of its original cradle"). Now, unless the edifice after its construction is to be turned upside down, it seems that, if egoism continues as the foundation, the roofing must be proportioned to the basement. Egoism must (tend to) reduce sympathy at least to the limits within which a clear sacrifice of personal happiness is not required by sympathy. And this is an intelligible and tenable, if not sublime, position; egoistic volition conniving at a limited exercise of sympathetic fixed idea, as being ineradicable and within limits conducive, or not fatal, to the pleasure of the agent, as necessary and useful, though "morbid and irrational[1];" very much as the pious Augustine contemplates the utility of a social evil. But if more is claimed for the fixed idea, if it is enthroned side by side with, or above volition, according to what laws will it rule? How will Mr. Bain's position differ practically from intuitivism? Will not the autonomous fixed idea be peculiarly open to the sarcasm directed by Mr. Bain against the Sidgwickian non-hedonistic preference[2], that it is as likely to lead to bad as good.

Ambiguous also is the position, in Mill's system, of the fixed idea; the very admission of which[3] appears to compromise the hedonistic origination of action so vigorously asserted in the "Proof" of "Utilitarianism." In the Autobiography, in the deeply interesting passages relating to Mill's despondency and its remedies, he may seem to recommend, as a means to personal happiness, the pursuit of others' good by way of fixed idea[4]. Lastly, if for Mill the only principle independent of, and therefore capable of being "paramount over, all selfish objects of desire[5]," is the fixed idea, what other origination has he for the enthusiasm of humanitarian religion—the religion of Mill?

[1] "Mental and Moral Science," *sub voce* Fixed Idea. [2] "Emotion and Will," third edition. [3] "Dissertations and Discussions," Review of Mr. Bain. [4] p. 142, &c. [5] Essay, p. 109.

Printed by James Parker and Co., Crown Yard, Oxford.

ERRATA.

p. 45, last line, *for* "Since, then," *read* "For as, then."
p. 48, line 1, *for* π_0 *read* $\pi_0 - cy_0$.
p. 49, lines 15 and 32, *for* dy^2 *read* δy^2.
p. 50, line 26, *for* $x_1 \ldots\ldots x_1$ *read* ${}_1x_1 \ldots\ldots {}_nx_1$.
p. 51, line 1, *for* the first two symbols, *read* ${}_1x\ {}_2x$.
p. 53, lines 12 and 14, *omit* $\times\ \phi'(x_1 - x_0)$.
Ibid., line 13, *for* + *read* −.
p. 54, note [2], *for* "b" *read* "and."
p. 59, line 22, begin a new sentence with "Or perhaps."
p. 62, line 17, *omit* +,
p. 69, line 25, *omit* "the normal ones and."
p. 74, *for* the last sentence *read* "And there is reason to suppose that, when y is small, this fraction is small."