# Samuelson's misses: A rejoinder\*

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#### 1. Introduction

Readers will probably have joined me in being disappointed at the lack of stringency in the Reply to my *Samuelson on Sraffa and the Classical economists*—henceforth, respectively, Samuelson (2007) and Garegnani (2007)—published jointly in the last issue of this Journal. The antipathy for adversary procedures in scientific discourse advanced by Samuelson (2007: 243) has in effect resulted in a failure to comment on the specific criticisms of his work raised in my article. Thus, in the course of this Rejoinder we shall notice that no answer is given to my rebuttal of the necessity of constant returns for the validity of Sraffa's and the classical economists' theory of prices and, connectedly, to my denial of a dependence of their theory on demand and supply on neoclassical lines.

But also missing are the answers on two other basic issues raised in my article. The first concerns the classical analysis of non-wage income in terms of the difference between the output and the consumption necessary to its production – in terms, that is, of the social surplus often indicated as a characteristic feature of Ricardo, and the classical economists: the question at the basis, for example, of Sraffa's interest in a "standard commodity", against which Samuelson has often argued (section IV in my 2007). The other issue regards the classical economists' theory of wages, the heart of their analysis and of its structure, as I have contended (Garegnani 2007,

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section I and II) and textually supported by the numerous well known 'puzzles' which Smith and Ricardo's theory of wages raise for modern interpreters (Garegnani 2007, section Vc), or by my criticism of Samuelson's interpretation of chapter XXXI 'On Machinery' of the *Principles* (Garegnani 2007, section Vd).

However the right spirit of scientific discourse does emerge when, in the Reply, we read that: "if my [...] syllogisms are found to be erroneous the present exercise will have been valuable in helping establish where the truth does probably reside" (Samuelson 2007: 244, italics in the original), a statement to which I would commit myself too, while taking it in the broadest sense suggested by the weighty word "truth".

## 2. The other paradigm of economic theory

The argument of my article had a twofold aim. The first and central one was to contend the existence in the early classical economists—and in Sraffa's (1960) *Production of Commodities*—of a theoretical approach to distribution and relative prices alternative to the dominant one ultimately founded on the substitutability between factors of production. My second aim was to recall the present state of the critique of this latter theory that has accompanied the resumption of the classical approach. That second aim was suggested by the 'doom' Samuelson (2000: 115 ff.) had predicted for the critique.

On the first point, the Reply is laconically unrepentant. Indeed, by means of its 'Master production function' it focuses on enlarging on that interpretation of Sraffa and the classical economists in terms of a 'discontinuous' version of neoclassical demand and supply, which I had briefly rebutted (Garegnani 2007, para. 24), as contrasting with the evidence we find in those authors.

On my contention that what we trace in their writings is instead the alternative approach to distribution and prices outlined there, the Reply states: 'It is a myth that there ever did exist a plausible classical paradigm in which competitive price ratios [...] were invariant to changes in objective consumers demand' (Samuelson 2007, p. 2). But in my article, what I deny the existence of in the classical economists is not that variance: it is the neoclassical interpretation of it advanced by Samuelson. As argued in para. 1 and 32 of Garegnani (2007), the latter interpretation depends, among other things, upon the assumption of a full employment of labour: it accordingly clashes with Ricardo's and Smith's admission of permanent labour unemployment. Connectedly it clashes with their wage theory,

so remarkably and significantly indigestible to modern interpreters (Garegnani 2007, para. 28-31). The Reply is silent on these contradictions to its interpretation.

We also read in the Reply:

'It is textually dubious that post-1870 neoclassicism [...] differed from classicism [...] importantly because the former linked purely competitive supply and demand with constant returns to scale, whereas the earlier group definitely did not have to do so.

(p. 2)

But no answer is given to my basic contention of a classical separate determination of outputs, which are then implicitly treated as 'intermediate data' in determining prices and to the corresponding textual evidence. This treatment makes constant or variable returns irrelevant in price determination, as distinct from their centrality in the analysis of capital accumulation and growth (Garegnani 2007, para. 5-7). Outputs as 'intermediate data' in price determination are in fact essentially what Samuelson himself attributes to Ricardo when he blames him for taking as given the position of the no rent land in his theory of relative prices (e.g. Samuelson 1998: 462ff). And, more generally, this Classical difference from later theory on returns was certainly important enough to compel Marshall to attribute to Ricardo an absolutely rigid demand for 'corn' and, somewhat contradictorily, constant returns for all other commodities (Garegnani 2007, para. 6). This separate determination of outputs was indeed the simple, general way out, which, owing to their different wage theory, the Classical economists could take, and instinctively took, out of what the Reply itself calls the 'quagmire' of returns in the theory of value (Samuelson 2007: 247, n. 3).

On the other hand, the above treatment of outputs as 'intermediate data' provides the answer to the Reply's alleged 'impossibility of natural classical prices' (Samuelson 2007: 248), due to that 'until economists Smith and Ricardo know *Distribution* they generically cannot know Values' (Samuelson 2007: 247, italics in original). This may seem surprising in a Reply to Garegnani (2007), where the difference between the two theoretical approaches is traced to their different theory of distribution. Indeed for the particular problem to which the Reply refers in the passage above—i.e. the necessity of consumer demand for knowing the ratio between the wage and the rent of land on which competitive normal or natural prices depend—I had argued (Garegnani 2007, Section I) that, given a real wage determined by its own circumstances, the separate determination of those outputs, and therefore their widely recognized treatment as givens, allowed

Ricardo—and by a more rigorous procedure, Sraffa—to determine rents and their ratio to wages.<sup>1</sup>

In conclusion, the Reply does not appear to answer my central contention, namely, that the theory of wages of Smith and Ricardo, generally admitted to differ from the modern one, does in fact entail a different determination of outputs and, thereby, an altogether different content and structure for the theory—in which, for example, supply and demand take on their original Smithian meaning of single quantities, not functions, relevant for the analysis of the gravitation of actual prices towards natural or normal levels, which are determined separately from those two quantities (Garegnani 2007, para. 8).

### 3. The critique of the dominant paradigm

With regard to the second, more critical aim of my article, the Reply is silent on my contention that the 'Hicksian' or (as more commonly indicated) neo-Walrasian reformulations of the theory (Garegnani 2007, para. 35) are emerging to be as dependent on the inconsistent conception of capital as a single magnitude as the traditional pre-Hicksian formulations were (Garegnani 2007, para. 37). It therefore also emerges that the same deficiencies, which allowed the early 1960–70 stage of the capital controversy to dismiss in a few years the formulations given by Marshall, Clark, Wicksell, Pigou, Robertson—and, in effect, by all other mainstream authors until those decades—also undermine those contemporary reformulations. And, the reformulations in question are those to which, as we shall presently contend, the Master production function of the Reply does in fact pertain.

What we find is only an indirect reference to the critique, when the Reply rejects 'the view that only stationary states [i.e., it appears, the 'normal positions' of Garegnani 2007, para. 34] are deductively tractable' (Samuelson 2007: 4). It is essentially the rejection of the traditional normal positions in favour of the Hicksian (neo-Walrasian) temporary or intemporal equilibria—a rejection imposed by the abandonment of the conception of the given capital endowment as a single magnitude on which the normal position has to rest in neoclassical theory in order to allow for the required adjusted physical composition of its capital

<sup>1</sup> As for the final influence of the Reply's consumer demand on this distribution, we already saw the question in connection with Samuelson's first passage above.

endowment—i.e. for its (riskless) uniform effective<sup>2</sup> rate of return on the capitals' supply prices<sup>3</sup> necessary, I have argued, for its 'persistency' and therefore its potential correspondence with observation (Garegnani 2007, para. 36).

Samuelson's passage above continued with: 'My use here of twenty-first century dynamic Samuelson Etula Master Function will rebut that claim', where the reference to a dynamic analysis may be intended to answer, in particular, the Marshall passage I used for my argument (Garegnani 2007, para. 34): 'dynamical solution in the physical sense of economic problems are unattainable' (Marshall 1898: 39), where, however, Marshall was surely not referring to any absence of abstract tools like the Master function for such solutions. What he judged unobtainable was a dynamics with the possibility of correspondence with observation that we can have in the physical sciences (Marshall's immediately preceding example was astronomy). In economics, he in fact continued, one has instead to be content with 'such rude and imperfect approaches to dynamical solutions as we may be able to attain to', i.e. essentially, the comparisons of the normal positions (Garegnani 2007 para. 36). But in neoclassical theory, and here comes the central point, this would require a consistent notion of the capital endowment as a magnitude susceptible of changing its physical 'form', though not 'its quantity'.<sup>4</sup>

In fact, the Reply provides no argument for its rejection of the normal position (and we shall see in Appendix 1, how some misunderstandings appear to underlie Samuelson's position on the matter). I can here, therefore, only refer the interested reader back to Garegnani (2007, para. 35) or to (Garegnani 1976) for a discussion of the issue.

<sup>2</sup> By 'effective', we mean the single rate of return on capital which underlies its multiple 'nominal' expressions dependent on the numeraire, occurring when future changes in relative prices enter the equilibrium conditions.

<sup>3</sup> The condition can be stated, if we prefer, as the equality between the demand prices of the capital goods and the respective supply prices: (cf. Appendix 1, p. 584, below. The capital goods referred to are of course always those pertaining to the dominant techniques).

<sup>4</sup> As Hicks put it *a contrario* when—still basing his analysis on normal positions and, therefore, on capital as a single magnitude—he described as follows what he called the 'short period marginal product of labour': 'the additional production due to a small increase in the quantity of labour *when not only the quantity, but also the form of the co-operative capital is supposed unchanged*' (Hicks 1932: 20, our emphasis) then dismissing the whole notion by: 'It is very doubtful if the conception can be given any precise meaning which is capable of useful application' thus adopting a marginal product of labour in which the 'quantity of capital' is constant, while its physical 'form' changes.

# 4. The Master production function

As for the Reply's own argument, I can say that I very much welcome its Master function—once, of course, it is ascribed where it essentially pertains, namely, to the contemporary reformulations of neoclassical theory and not to the Classics or Sraffa, who do not know, for example, of the equality postulated there between factor demands and factor supplies, or indeed of any such 'demands' and 'supplies'.<sup>5</sup> I do in fact welcome the Master function because of its dedication to pinning the little of factor substitution remaining in the Hicksian reformulations of the theory down to earthy numerical examples. Indeed, once the capital endowment is expressed as a physical vector, as is done there, it should be evident that, for the components of the vector, the continua of alternative methods of the traditional production functions cannot exist and that, accordingly, only the finite number of alternative methods of the Master function has any plausibility left.

As will be argued in Appendix 1, the Master function does indeed highlight an element that, perhaps because of its less strictly analytical nature, has gone unduly unstressed in the course of the capital controversies so far. The function makes, so to speak, tangible the near absence, in those reformulations, of the factor substitutability on which the theory must however continue to rest for its validity—a third main difficulty besides the emerging continuing dependence on 'capital', the single productive factor and the methodological one recalled above concerning correspondence with observation. Like its near homonym, Samuelson's (1962) own Surrogate production function of old, the Master production function might therefore end by making the inherent weaknesses of contemporary theory more evident.

Some question begging seems indeed to underlie the optimistic contrary opinion, struck in the conclusion to the Reply, when we read: 'a pro-Sraffian who believes strongly that the world empirically has but few viable alternative subtechnologies ought to expect distributive shares to jump around volatiley in a way that econometricians do not find to occur' (Samuelson 2007: 262).

<sup>5</sup> In connection with this attribution we also read in the Reply (p. 22): 'Sraffa (1960, part III) went some limited steps towards seeking missing equations along the lines [of the Master function]' but in Sraffa we find the coexisting methods of the Master function when distribution between wages and profits happens to allow for them, with the distribution, therefore, determining their coexistence and not vice versa.

But the dispute is about validity of the theory, and its contradictions with reality are clearly an element for the critique of the theory and for an alternative to it, not for its posteriori validation. The passage constitutes, indeed, as clear a confirmation as we may wish of my general point (Garegnani 2007, para. 1, 8, 23, *passim*) about Samuelson's tendency to take neoclassical demand-supply as facts and not as a theory, subject to disproof like any theory.

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#### Appendix 1: On the 'Master function'

#### The Master function and factor demands

For my purpose here, I must first give a brief account of what I understand to be the Master function of the Reply. The function supposes the existence of a discrete number of alternative methods for the (direct and indirect)

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production<sup>a</sup> of goods, which the consumers are assumed to demand in fixed proportion (apparently in order to reflect the absence of utility analysis in the Classical authors to whom the function is ascribed).<sup>b</sup> The quantity of labour available is taken as a unit so that both outputs and the remaining factor endowments—which the Reply's Table II takes to consist only of the several capital goods—are directly measured as proportions to labour.

Neoclassical competitive interaction between consumer demand and factors supply governed by given endowments would then lead for an equilibrium to what we may call 'cluster' of such methods sufficient to define by their coexistence the price position of the economy (think, for a simple example, of a labour and land only economy producing corn and cloth, where the method of production of cloth plus two methods for producing corn on homogeneous land suffice to determine wage, rent and cloth-to-corn price). Given a sufficient number of factors (in particular capital goods) common to the coexisting methods of each 'cluster', changes in the proportions in which the latter are activated, define, 'areas' of factor endowments (the factors are assumed to be rigidly supplied), which could be fully employed with the methods of the 'cluster' in question (cf. Samuelson 2007, Figures 1 and 2), so that any endowment in the 'area' would be compatible with equilibrium at the prices defined by the 'cluster'. Looking at it from the side of each factor, the proportionate changes in the activation of the methods would describe a horizontal segment in its demand, where equilibrium for the system could be achieved if the endowment of the factor happened to fall in that segment.<sup>c</sup>

- a The Reply seems to take the Master function as including the traditional continuous production function (Samuelson 2007: 9–10), so that also in a context of discrete alternative methods we should take the function as a single one taking as many forms as there are 'clusters' of methods. Here I shall refer exclusively to the discrete case on which the Reply is in fact focused.
- b Nothing, however, prevents us from dropping that assumption and letting the proportion of the goods demanded by consumers depend on their relative prices: when we refer to the Master function as pertaining to the contemporary reformulations of the theory, we evidently envisage such a completion of the Reply's treatment of the function (see also footnote d).
- c This factor demand segment, as well as the whole step demand function of which it is a part, are traced on the assumption that the remaining factors in the endowment are fully employed and, more generally, that equilibrium holds in all markets except the one of the factor in question (the income of the owners of the factor is accordingly assumed to correspond to the quantity demanded and therefore actually sold by them, and not to the quantity supplied at the assumed prices. As may be easily realized this makes a disequilibrium in the single market of the factor in question compatible with Walras' law: (cf. the notion of 'general equilibrium demand function' in e.g. Garegnani 2000: 201, n.15)).

If the available alternative methods allow for several such 'clusters' and they are also such as to have a sufficient number of capital goods in common also between the clusters, then we shall have for each factor a decreasing step demand function, each horizontal segment corresponding to one of the clusters. Prices between those of the clusters, i.e. between those corresponding to the horizontal demand segments for each factor, would be indeterminate: but this indeterminacy would be bounded by the adjacent horizontal segments and it could only occur for factor endowments lying exactly on the edge between two endowment 'areas'.<sup>d</sup> That indeterminacy would therefore mean, above all, that prices move in jumps as changes in tastes or endowments shift the economy from one cluster to another.

#### The Master function's marginal products

The Reply however does not explicitly consider the above demand functions. It focuses instead on the equivalent fact that—given a sufficient number of capital goods common between the methods forming each cluster, and between those of the several clusters—a key assumption for the whole construct—then the methods of each cluster will make it possible to define what Samuelson envisages as 'marginal products' of the factors. They are obtained by changing the proportions in which the methods are activated so as to keep constant the quantity of all factors but the one in whose 'marginal product' we are interested. By construction, the result will coincide with a horizontal segment of the above step demand of the factor.

It should be noted straightaway, however, that the 'marginal products' resulting from the Master function are different from the traditional ones, founded on intellectual experiments of the individual producers. Thus, their equality with the respective factor price is not a direct entailment of producer maximization: it is simply a result of the competitive price

d It may however be noted that the vertical segment would not be such if we introduced—as we saw the Reply makes a point of not doing—a physical composition of consumption which would change, and change factor demands accordingly, as commodity and factor prices move from cluster to cluster. This however will again be so only if the methods of production of the alternative consumption goods had a sufficient number of capital goods in common. This indirect substitution operating through consumer choice will accordingly be subject to limitations parallel to those we shall see below for the factor substitution operating through alternative techniques.

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equations, corresponding to the methods of the clusters.<sup>e</sup> These equations, it is then important to notice, would be sufficient to determine the price position of the economy, even when capital goods differ between methods enough as not to allow for any such marginal products.<sup>f</sup> Now, this dependence on the price equations is in contrast with what happens for the traditional marginal products which, since they entail no coexistence of methods, do add conditions to the price equations of the products, whose number would otherwise be insufficient for the determinacy of the price system. The difference between the two kinds of marginal products is, on the other hand, implied by the Reply itself, when it is remarked that the marginal products of the Master function are 'locally linear' (Samuelson 2007: 25), i.e. that (i) they are linear within each cluster and (ii) the function giving them must be redefined for each such 'cluster'. Both such properties are alien to the traditional marginal products.

#### A first limitation of the Master function

I take the essential analytical point of the Master function to lie in exhibiting, in a concrete way, factor substitution such as is possible in the Hicksian reformulations of the theory (section 4 of the text above). Two observations seem however sufficient to indicate how the Master function helps us to realize the strict limitations of that substitutability. The heterogeneity of the capital goods between alternative production processes—which deprives any 'smooth' factor substitutability of sense, even as an approximation, as a factor—also drastically reduces the discontinuous substitutability analysed in the Master function.

The first such observation, which has been aired already in this Appendix, is that there is no reason why the methods of the clusters

e Since the constant prices of the 'cluster' must equal costs, a change in the proportions of the alternative methods allowing for a unit increase in the employment of the variable factor, will result in an increment of the value of the aggregate output which is necessarily equal to the service price of the factors. And when that value increment is referred to a single commodity, that service price expressed in terms of that commodity will have to equal the physical increment or 'marginal product' of the factor.

f See, for example, the case of the techniques assumed by Samuelson in his (1962) Surrogate capital article such that a consumption good is produced by labour and a capital good, specific to the technique, in turn produced by itself and labour. Each couple of such techniques can form a cluster determining at least one series (cf. note h below) of real wage, interest rate, and the consumer good prices of the two capital goods specific to each techniques. These clusters will not allow for a Master function and its 'marginal products'.

should have in common a sufficient number of kinds of capital good, as is assumed in the Reply. Indeed, elementary observation suggests that this is not (relevantly) the general case. But then, not only the Reply's Master function and its marginal products will not exist, but it will be difficult to envisage factor demands leading to plausible non zero prices for the factors (whose scarcity Samuelson seems here to correctly take as known from experience rather than as results of the equations).<sup>g</sup> The possibility of a reswitching of techniques will then also arise and it will no longer be true even that a given cluster of techniques defines only one economically significant price positions of the economy.<sup>h</sup>

#### A second limitation of the Master function

The second observation is that even when, as in the examples of Table II in the Reply (p. 19), the number of capital goods common to the methods are sufficient for step demand functions (and 'marginal products') of the factors, even then the possibility of substitution would generally be very limited. This is well exemplified by the mentioned clusters of Table II, where the change in the composition of consumer demand from 'wheat only' to 'wheat and iron', illustrated by moving between the two sets of quadrangles of Fig. 2 in the Reply (p. 260), shows that none of the endowments ensuring non-zero factor prices for the case 'only wheat', does the same for the case 'wheat and iron'.

To avoid consumer choice leading, in this way, to zero prices of the factors of the initial endowments, in particular zero gross prices of capital goods services, the endowment 'areas' compatible with full employment under different compositions of consumer demand should broadly coincide, as it was possible to suppose in the case of the traditional continuous production function based on 'capital', the single magnitude supposedly susceptible to changing its 'form'.<sup>1</sup> The Master function then makes us 'touch', so to speak, how difficult it is to satisfy that condition, when adopting a vectorial notion of the capital endowment.

g 'Relevantly' in the sense that changes in the proportions of the factors thus physically defined, when at all possible, are likely to be dominated at the corresponding new prices by alternative methods requiring changes in the kinds of capital goods employed.

h Cf. the clusters of two techniques of the 'Surrogate capital' kind mentioned in footnote *f* above. A reswitching between two such techniques (cf. e.g. Garegnani 1970: 413, Fig. 2) entails of course two points where the techniques coexist, thus defining two series for wage, interest rate, and the prices of the two capital goods.

i Cf. the Hicks (1932) passage quoted in n.4 of the text above.

#### A missed implication of the Master function

We have so far seen the Master function as a tool for the analysis of factor substitution in what are in effect the contemporary reformulations of neoclassical theory. But Samuelson's treatment of the Master function also brings to light in a rather striking way what was listed in section 4 of the main text as, the second basic deficiency of contemporary neoclassicism besides the emerging continuing dependence on a 'quantity of capital': namely, the undermining of the possibility of correspondence between theory and observation.

As indicated in an old comment of mine (Garegnani 1976) on Samuelson's work, by expressing the capital endowment as a physical vector Samuelson was bound to refer to a position of the economy where the effective rates of return<sup>j</sup> on the capitals' supply prices do differ—to refer, that is, to a position that the economy moves away from, rather than tending towards, just as it would move away from a position in which wages for the same quality of labour happened to differ or prices of commodities happened to differ from the respective supply prices.

Samuelson, however, incorrectly labels that divergence of effective returns as it emerges, e.g. from the Table II cluster *b*, *c*, *A*, *B*, corresponding to endowment  $\beta$  (Samuelson 2007: 260), as a divergence of 'commodity own rates of interest' (p. 261n), a phenomenon that is instead the mere result, when it occurs, of considering within the equilibrium conditions future changes in relative prices<sup>k</sup> (of which there is in fact no trace in the Reply).

This incorrect labelling, then, appears to lead Samuelson into an incorrect determination of the corresponding equilibrium. While the divergence of own commodity rates of interest in no way affects the equality or inequality between the price of a commodity and its costs or supply price, a divergence of the effective rates of return on the capitals' supply prices prevents that equality for the capital goods giving a lower such return than one or more others. The nature of savings as demand for the single commodity 'future income' (cf. e.g. Garegnani, 2000, para. 12, 22–3) is indeed such that, as is on the other hand well known, arbitrage imposes the corresponding single price, i.e. the effective rate of return on the supply

j For the adjective 'effective', attributed to that rate of return see n.2 of the text above.

k If a commodity *A* is going to rise in price by 1% relative to *B*, over the year of the loan, arbitrage will evidently tend to impose for a loan denominated in *A* an 'own' interest rate, which is 1% lower than that for a loan denominated in *B*. This divergence clearly has nothing to do with capital goods in particular, and even less with the returns over their supply prices resulting from the Master function.

price of the capital good having the highest such return, also on the remaining capitals by lowering their 'demand price' sufficiently below the respective supply price.<sup>1</sup> Now, we find no trace of such a lower demand price in the case of equilibrium  $\beta$  where prices remain equal to their supply prices and, accordingly, iron's effective rate of return is 1.72% vs. 13.55% for wheat (Samuelson 2007: 261n), a situation that no arbitrage would allow in the economy.

This incorrect determination of capital goods' prices has, in turn, consequences because the capital goods with a demand price below supply prices will evidently not be reproduced and the physical net product of the Master function will be entirely notional. The impermanence that prevents such equilibria from being a possible basis for correspondence with observation does then appear here in the 'tangible' fact that the capital endowment will soon have to change, so as to allow, for example, for iron production and, hence, at one remove, wheat production as well to occur. And with it Master function and price system will also have to change.

l See e.g. Walras' distinction between 'prix de vente' and 'prix de revient' of the 'capitaux proprement dits' (1954 e.g. 2771 ff).