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## A Critique and Reinterpretation of Marx's Labor Theory of Value

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### I. INTRODUCTION

The three central issues of classical political economy are: first, the nature of real wealth; second, the causes and instrumentalities of economic growth; and third, the determinants of the distribution of real wealth among the three great classes of society. The first issue was settled by Smith, who gave to classical theory the definitive statement of the thesis that real wealth is physical goods (and services, although Smith did not emphasize them), not gold and silver money. The second and third issues, Smith argued successfully, could be handled only through an understanding of the institution of the market. In a capitalist economy, such an understanding requires a theory of market exchange under conditions of competition, in which each entrepreneur seeks the highest possible return on the value of the capital invested. Since the physical wealth of the economy is distributed to workers in the form of wages, to landlords in the form of rents, and to capitalists in the form of profits, and since the magnitudes of these distributive variables, as equivalents of real wealth, are in turn determined by the prevailing system of prices, it followed that the theoretical center of a satisfactory political economy must be a theory of prices. In the eighteenth-century language which became standard for the subject, what was required was a *theory of value*.

Drawing on the successful model of natural science, Smith proposed as the appropriate object of investigation for political economy *natural prices*, which, as he put it in a famous and oft-quoted passage, are "the central price(s), to which the prices of all commodities are continually gravitating" (Bk. I, chap. 7). Natural prices are contrasted

with market prices, the prices at which, on particular occasions, commodities actually exchange. The deviation of market from natural prices forces the rate of return to particular capitalists above or below the natural rate of return, and thereby motivates movements of capital. The result is an equilibration of the society-wide profit rate through readjustments of the size of output in particular industries, through the taking up of more profitable techniques of production and the dropping of those less profitable, and so forth.

Smith argues that in "that early and rude state of society which precedes both the accumulation of stock and the appropriation of land," commodities will exchange in proportion to the quantity of labor required for their production (labor being measured in units of time). But he is aware that the existence of stock and privately owned land irreversibly distorts exchange relationships, and lacking anything resembling a *theory* of price determination, he offers instead a simple description—the so-called adding-up account, according to which the price of a commodity is the sum of the (natural) wages paid to labor, the (natural) rents paid to landlords, and the (natural) profits paid to capitalists.

Ricardo arrives at the first coherent and powerful theory of natural prices (and with it, a theory of distribution) by means of two great theoretical advances, the first owing really to the work of Torrens, West, and Malthus, the second his own contribution. The first step is to demonstrate that rent plays no role in the determination of price, but rather is a portion of profits appropriated by landlords at the expense of capitalists. The key to the argument is simply that in a condition of scarcity of land, in which demand causes food producers to bring less and less fertile land into cultivation, the natural price will be the same for grain produced on the most fertile land and grain produced on that land so infertile that producers can afford to pay only a vanishingly small rent if they are to make the going rate of profit on their investment. Since rent does not appear in the equation representing the determination of the price of that "marginal" output (*not* the same notion later employed by the marginalists—the word "marginal" does not appear in Ricardo's collected works), it follows that rent plays no role at all in the determination of price.

With rent disposed of, Ricardo concentrates on the real stumbling

block in the way of a satisfactory theory of natural price: the role of stock, which is to say, capital other than outlays for wages. Ricardo's great theoretical breakthrough, of course, is the notion that the exchange of commodities is regulated *not* by the quantities of labor directly required for their production, but rather by the quantities *directly and indirectly* required for their production—where “indirectly required” simply means “required for the production of some non-labour input directly required, or directly required for some labour or non-labour input which in turn is required for the production of some non-labour input directly required, . . . and so forth.”

If we assume, as Ricardo did, an economy consisting of  $n$  one-product industries, with one technique of production for each sort of commodity, with a single quality of labor, and (for the moment, and for simplicity) only production of commodities which are themselves directly or indirectly required for the production of all other commodities, then we can easily enough solve the problem of determining how much labor is required, directly or indirectly, for the production of a unit of each good.

Let the structure of the economy be as follows, where in general  $I_J$  is the amount of commodity  $I$  used as input in the industry producing commodity  $J$ ,  $O_I$  is the output of commodity  $I$  in the industry producing  $I$ , and  $L_I$  is the quantity of labor directly required in the industry producing commodity  $I$ :

$L_A, A_A, B_A, C_A, \dots, N_A$  yields  $O_A$

$L_B, A_B, B_B, C_B, \dots, N_B$  yields  $O_B$

.....

.....

$L_N, A_N, B_N, C_N, \dots, N_N$  yields  $O_N$

We assume, of course, that  $O_A$  is equal to or greater than  $(A_A + A_B + \dots + A_N)$ , and so forth for each output. In order to insure a positive rate of profit, we assume as well that at least one of the outputs is strictly greater than the sum of the inputs of that commodity.

If we divide the description of each industry through by the magnitude of output in that industry, we arrive at the quantities of labor and non-labor inputs required for a single unit of output—what are usually called the unit input coefficients. Let us now define a set

of  $n$  variables,  $\lambda_1$  through  $\lambda_n$ , each of which represents the quantity of labor directly or indirectly required for the production of a single unit of the commodity indicated by the subscript. (These, of course, are what have come to be called "labor values," but it is important to keep it fixed in our minds that "labor values" are merely quantities of labor directly or indirectly required for the production of unit quantities of various commodities.) Now, since the labor inputs are quantities of labor directly required, by the production processes, whereas the non-labor inputs represent, or arise out of, quantities of labor indirectly required in the production process, we can transform our description of the economy into a system of  $n$  equations in  $n$  unknowns (namely, the  $\lambda_i$ ). Let us use the familiar notation  $a_{ij}$  = the quantity of commodity  $j$  required in the production of one unit of commodity  $i$ . Then our description of the economy translates into the following system of equations:

$$\begin{aligned} l_1 + a_{11}\lambda_1 + a_{12}\lambda_2 + a_{13}\lambda_3 + \dots + a_{1n}\lambda_n &= \lambda_1 \\ l_2 + a_{21}\lambda_1 + a_{22}\lambda_2 + a_{23}\lambda_3 + \dots + a_{2n}\lambda_n &= \lambda_2 \\ \dots & \\ l_n + a_{n1}\lambda_1 + a_{n2}\lambda_2 + a_{n3}\lambda_3 + \dots + a_{nn}\lambda_n &= \lambda_n \end{aligned}$$

Using familiar matrix notation, where  $l$  is a column vector of direct labor inputs,  $A$  is an  $n \times n$  matrix of unit input coefficients, and  $\lambda$  is a column vector of labor values, the system can be represented by the matrix equation:

$$(1) \quad l + A\lambda = \lambda$$

If  $A$  is indecomposable, which is to say that each commodity is directly or indirectly required for the production of all others (that they are all "basics" in Sraffa's terminology), then the matrix  $[I - A]$  is non-negatively invertible, and we can therefore solve the equation for the vector of labor values,  $\lambda$ , giving us:

$$(2) \quad \lambda = [I - A]^{-1} l. \quad \lambda > 0$$

Ricardo's claim is that commodities exchange in proportion to the quantities of labor directly or indirectly required for their production. This, he says, is their *natural* price. Market price will, of course, fluctuate about this center of gravity. (Cf. Ricardo's *Principles*, chap.

IV, where he accepts Smith's treatment of the subject *tout court*.) But economic agents do not buy and sell according to labor values. So, it is necessary to prove that commodities exchange in proportion to the quantities of labor directly or indirectly required for their production. Formally, this means solving a system of price and profit equations, and then demonstrating that the vector of prices that solves the equations is unique, positive, and proportional to the vector,  $\lambda$ , of labor values.

The price equations for the sort of system we are examining are the following:

$$(3) \quad (lw + Ap)(1 + \pi) = p, \quad \text{where } w \text{ is the money wage, } \pi \text{ is the uniform rate of return on the value of capital invested, and } p \text{ is the vector of prices.}$$

This is a system of  $n$  equations in  $(n + 2)$  unknowns: the  $n$  prices,  $w$ , and  $\pi$ . The system thus is underdetermined, with two degrees of freedom. The number of unknowns can be reduced by one by the device of selecting one price arbitrarily as numeraire, and setting it equal to 1. This has the effect of converting the other  $n - 1$  prices into relative prices, which is all we need or wish to know anyway. In order to reduce the system one degree further, it is necessary to fix a second variable. Since in a system of free competition it is not meaningful to fix a relative price or the rate of profit arbitrarily, the system can be made determinate only by specifying the wage. In classical political economy, this is done by assuming a fixed real wage (that is, a fixed physical market basket of goods per unit of labor), represented by an  $(n \times 1)$  column vector,  $b$ , of goods per unit of labor ( $b$  for *basket*). It follows that the inner product of the real wage vector and the price vector is equal to the money wage, which is to say:

$$(4) \quad w = bp.$$

Substituting in (3), we then get:

$$(5) \quad (lb p + Ap)(1 + \pi) = p$$

Now,  $l$  is an  $(n \times 1)$  vector, and  $b$  is a  $(1 \times n)$  vector. Hence,  $lb$  is an

$n \times n$  matrix representing the non-labor inputs, per unit of commodity output, required for the labor inputs into the  $n$  industries. (It is assumed that workers do not spend any of their wage on labor services, an assumption that fits nicely with the independent assumption of a subsistence wage. In the model proposed later in this paper as an alternative to the standard Ricardian/Marxian model, this unrealistic assumption is eliminated.) By factoring out  $p$ , and then adding  $lb + A$  to form the "augmented unit input coefficient matrix,"  $A^*$ , we are in a position to rewrite equation (5) as:

$$(6) \quad A^*p (1 + \pi) = p$$

This is an eigenequation which, given the properties of  $A^*$ , has a unique all positive solution,  $p$ , associated with a positive rate of profit,  $\pi$  (that is, the maximum eigenvalue of  $A^*$  is  $0 < \frac{1}{1 + \pi} < 1$ ).

The vector,  $p$ , of prices is unique and positive, but unfortunately for Ricardo, it is not in general proportional to the vector,  $\lambda$ , of labor values. (A fact which Ricardo himself recognized, of course. See the draft essay, "Absolute Value and Exchangeable Value," on which Ricardo was working at his death in 1823. *Works*, edited by Sraffa and Dobb, vol. IV.) What is true is that if the proportion of labor directly required to labor indirectly required is the same in all lines of production, then the vector of prices will be proportional to the vector of labor values. (The labor directly required, per unit output, in the  $i$ th industry, is  $l_i$ . The labor indirectly required is  $A_i\lambda$  where  $A_i$  is the  $i$ th row of the matrix  $A$ .) This condition is formally equivalent to the condition that there be what Marx later called equal organic composition of capital. The reverse implication holds so long as  $\pi > 0$ . The degenerate case of  $\pi = 0$  occurs when there is no physical surplus at all (strictly speaking, a possibility we have ruled out), or when the wage absorbs the entire surplus.

## II. MARX'S CRITIQUE OF RICARDO

Marx begins where Ricardo leaves off, with a recognition of the fact that prices are proportional to labor values only when there is equal organic composition of capital (leaving to one side the exotic cases

just discussed). Eventually, Marx believes himself able to solve the problem of the relationship between prices and labor values in a way that will simultaneously preserve the significance of the labor theory of value and reveal the mechanism by which the capitalist market mystifies the source of surplus value. (Marx, like Smith and Ricardo, seeks a theory of natural prices, not of market prices, although he has a good deal more to say than his predecessors about the dynamic processes by which capitalists, in responding to deviations of market from natural prices, take actions which cause the economy to grow and alter.) However, Marx perceives in the Ricardian theory a deeper problem—one which afflicts even the special case in which there *is* equal organic composition of capital. This problem, Marx believes, goes to the very heart of the classical theory of capitalism, and in Volume I of *Capital*, therefore, he *assumes* equal organic composition of capital so as to focus attention exclusively on the deeper issue, without the distracting complications posed by the deviation of prices from labor values. The problem Marx sees is this: Ricardo and the classical political economists have a theory of the causes of variations in the profit rate—in particular, they are able to show that wages vary inversely to profits, from which it follows that there is an ineradicable conflict of interest between workers and capitalists. But Ricardo is quite unable to explain why, in general, there are profits at all. On the assumption that equals are exchanged for equals in the marketplace, and on the further assumption that the natural price of any commodity is simply its labor value, it would appear that no capitalist could ever, by combining factor inputs, obtain an output which could be exchanged on the market for more than was laid out for its component inputs. In the words of Martin Heidegger, echoing a question asked three centuries earlier by Leibniz, *Warum gibt es überhaupt Etwas, und nicht Nichts?* (Why is there in general something [some profits] and not nothing?)

Marx poses the puzzle in Chapter V of *Capital*, entitled “Contradictions in the General Formula of Capital,” and offers his own solution in Chapter VI, “The Buying and Selling of Labour-Power.” It had already been established by Ricardo that the labor value of a unit of labor itself, like the labor value of any other commodity, was simply equal to the quantity of labor directly or indirectly required for the

production of that unit of labor. In the notation we have adopted, treating labor as the zero'th commodity, this meant that

$$(7) \lambda_0 = b\lambda$$

Marx argued (with bitter irony—the ironic structure of *Capital* is, in my judgment, indispensable to its theoretical purposes, and not merely a literary grace or an expression of Marx's personal anger. However, that is a dimension of the correct interpretation of Marx's theory which cannot be dealt with in this paper) that the capitalist could, in conditions of competition with goods exchanging in proportion to their labor values, earn a profit only if he could "be so lucky as to find, within the sphere of circulation, in the market, a commodity, whose use value possesses the peculiar property of being a source of value, whose actual consumption, therefore, is itself an embodiment of labour, and consequently, a creation of value." Fortunately, the "possessor of money" (*Geldbesitzer*, elsewhere translated beautifully as "Moneybags") "does find on the market such a special commodity in capacity for labour or labour-power."

Marx believed that the discovery of the distinction between labor and labor power, and with it the associated discovery of the concept of surplus value, was his major theoretical contribution to political economy. Every reader of *Capital* is familiar with the manifold ways in which Marx deploys both notions, as he analyzes the struggle over the workday, the introduction of machinery, and the process of capital accumulation. (For Marx's own view of what is best in *Capital*, see his letter to Engels of 24 August 1867, in *Werke*, vol. 31, pp. 326-327.)

The trick in the solution of the puzzle of profit, Marx claims, is that a distinction can be drawn between labor-power, or the ability to labor (= *Arbeitskraft*), and the activity of laboring, or labor (= *Arbeit*). Strictly speaking, what the worker sells is his or her ability to labor during a unit period of time, such as a day. The labor-value of one day's ability to labor is simply the quantity of laboring (measured, say, in hours of labor) required directly or indirectly to produce the food, clothing, and shelter necessary to restore that used up ability to labor (always allowing something for depreciation of capital stock, which is to say wear-and-tear on the human body, in the form of a



bit extra to raise a family). The labor value of a day's ability to labor is thus determined partially by the historical and social factors which define what will count as a subsistence wage, and partially by the "facility of production" of wage goods (to use Ricardo's terminology). So long as the economy is capable of producing any physical surplus at all, it will be the case that the labor value of a day's ability to labor is less than the number of hours of labor extracted from the worker in a workday.

It follows that in each line of production, the capitalist will end a cycle of production with goods embodying more hours of labor than were embodied in the inputs used up to produce that output. The extra or surplus labor value will have been extracted from the workers in the form of hours of laboring over and above what is socially necessary to reproduce their ability to labor, or their labor power, for another day. Taking the society as a whole, the labor value of the total physical surplus will exactly equal the surplus labor value extracted from the labor inputs. It is this extraction of surplus value that Marx labels *exploitation*. In order to see how this works in general, let us go back to our very first representation of a simple reproduction model. Let us now add the following notation:  $V_A$  will stand for the labor value of commodity A (that is, it stands for the quantity of labor directly or indirectly required for the production of a single unit of commodity A), and so forth.  $V$  will stand for the labor value of a single unit of that peculiar commodity, labor power. We can now write the following system of equations:

$$\begin{aligned}
 (8) \quad & L_A + A_A V_A + B_A V_B + \dots + N_A V_N = O_A V_A \\
 & L_B + A_B V_A + B_B V_B + \dots + N_B V_N = O_B V_B \\
 & \dots \dots \dots \\
 & L_N + A_N V_A + B_N V_B + \dots + N_N V_N = O_N V_N
 \end{aligned}$$

The value of labor power itself is given by an additional equation derived from the real wage, which we may represent as  $(A_w, B_w, \dots, N_w)$ :

$$\begin{aligned}
 (9) \quad & A_w V_A + B_w V_B + \dots + N_w V_N = L V \\
 & \text{where } L = L_A + L_B + \dots + L_N
 \end{aligned}$$

If we add both sides of the equation system (8) through (9), and simplify by means of the notational substitution  $A = A_A + A_B + \dots + A_N + A_W$ , and so forth for  $B, C, \dots, N$ , then we obtain:

$$(10) \quad L + AV_A + BV_B + \dots + NV_N = O_A V_A + O_B V_B + \dots + O_N V_N + LV$$

Rearranging, we have:

$$(11) \quad L(1-V) = (O_A - A)V_A + (O_B - B)V_B + (O_C - C)V_C + \dots + (O_N - N)V_N$$

The term on the right is simply the labor value of the physical surplus produced in the system. The term on the left is the surplus labor value extracted from each unit of labor input (that is,  $1 - V$ ) multiplied by the total number of units of labor employed in the economy as a whole ( $= L$ ). Q.E.D.

This striking equality, Marx claims, derives from two assumptions: first, that labor is the substance of value, so that the value of a commodity is determined by the quantity of labor directly or indirectly required for its production; and second that labor-power, or the ability to labor, can be distinguished from labor, or the activity of laboring.

### III. CRITIQUE OF MARX'S THEORY OF SURPLUS VALUE

There is no question that the second assumption cited at the end of the previous section is correct. There is no doubt, that is to say, that the ability to labor can be distinguished from the activity of laboring. The first assumption, however, is very much in question. Marx's argument for it, at the beginning of Chapter I of *Capital*, is extremely weak—so weak as not to constitute any argument at all. The *real* argument, I suggest, is the very solution of the paradox of profit which we are now examining. Like many other philosophers (among whom Kant and Hume spring to mind), Marx argues for his fundamental principle by claiming that with it, *and with it alone*, we can resolve a puzzle, solve a problem, answer a question, which previous theorists have been unable to handle. Since the algebraic equality expressed in equation (11) is very striking indeed,

it can, I think, be granted that *if* Marx can show that the existence of profit can be explained *only* by invoking the assumptions that labor is the substance of value and that labor power can be distinguished from labor, then he will have made a very persuasive case for the labor theory of value, at least in the special case of equal organic composition of capital. If Marx can then extend his account to deal with the general case of unequal organic composition of capital, he will have succeeded in establishing a theory of natural price, which is to say he will have laid the foundations for a satisfactory political economy.

By a slight alteration of the argument contained in equations (8) through (11), consisting of nothing more than a re-labeling of quantities and variables, it is possible quite easily to show that Marx was in fact wrong, and that neither the labor/labor-power distinction nor the assumption that labor is the substance of value is required in order to "explain" the emergence of profit.

Recall, first of all, that by "the labor value of a commodity," we mean nothing more nor less than the quantity of labor directly or indirectly required to produce one unit of that commodity. We can just as easily ask how much corn, or iron, or cloth is required directly or indirectly to produce one unit of each commodity. So long as each of these commodities is in fact directly or indirectly required for the production of every commodity, there will be an answer. (It is *not* necessary that the commodity in terms of which we are carrying out the calculation be directly required, as in fact labor is, for the production of each commodity. If labor is required directly in each line of production, for example, and corn is required for the production of labor, then corn will be directly or indirectly required for the production of each commodity, even if the production of labor is the only case in which corn is directly required!) Following the terminology used thus far, we may refer to the corn value, iron value, or cloth value of a commodity.

Now, reconsider equations (8) through (11). Let us re-label, so that labor is not distinguished from any other commodity. This has the effect of combining equations (8) and (9), since labor is no longer notationally identified. At the same time, we can relax the assumption, introduced earlier, that there are no direct labor inputs

into the production of labor power and that no surplus labor is produced. We will simply assume the existence of industries A through N, each of which may or may not use some of its own output as input and at least one of which produces a physical surplus of its product.

Choose any commodity arbitrarily, and let it serve as "substance of value." By this, I mean simply, ask how much of *it* is required, directly or indirectly, to produce one unit of each commodity including itself. Let us use the variables  $\overset{a}{V}_A$  through  $\overset{a}{V}_N$  to stand for those quantities of commodity A. They are thus variables standing for the A-values of commodities A through N. Now, carrying out the algebraic manipulations corresponding to those in equations (8) through (11), we arrive at the general equation:

$$(12) \quad A(1 - \overset{a}{V}_A) = (O_A - A)\overset{a}{V}_A + (O_B - B)\overset{a}{V}_B + \dots + (O_N - N)\overset{a}{V}_N$$

(For those who are checking the algebra, the trick is to subtract the quantity  $A\overset{a}{V}_A$  from each side of the equation.) *What (12) says is that the surplus A-value extracted from the A-inputs exactly equals the A-value of the physical surplus produced in the system as a whole. This result is obtained without assuming that labor is the substance of value, and without attempting to draw a distinction between A and A-power. So Marx is wrong.*

If we meditate for a bit on the algebra of this simple demonstration, it will become clear that the secret of the equality asserted in equation (12) lies in the fact that we have treated commodity A differently from the way in which we have treated commodities B through N. In the tallying up represented by the equations, the inputs of commodity A have been valued at one per unit—at par, so to speak—whereas all other inputs have been discounted by their A-values (that is, multiplied by the quantities of commodity A directly or indirectly required for the production of one of their units). Now, this method of computation certainly corresponds to what is meant by the phrase, "Quantity of A directly or indirectly required." But it does *not* correspond to the way in which a capitalist calculates the price of producing one unit of his output. He (it is, we may safely assume, he) will value

the  $A$ -inputs at their market price—which, we are assuming in this exercise, is their natural price. If one unit of  $A$  sells at  $\overset{a}{V}_A$ , then it will enter into his price calculations at  $\overset{a}{V}_A$ , not at 1.

The situation we face is this: in the logical space which we are investigating, all commodities, *including labor*, appear to have the same formal structure. The choice of labor as “substance of value” is arbitrary, and without significance, unless it can be shown that labor is in some way formally distinguishable from all other commodities. To say that it is formally distinguishable is to say that it, its production, and its role in the system of production, exhibit formal peculiarities, distinctive features of structure by means of which it can be identified and analyzed regardless of the conventions of notation. Marx thought he had found such distinguishing formal marks in the distinction between labor and labor power and in the algebraic equality of the surplus value extracted from the labor inputs and the labor value of the surplus. But Marx, as we have seen, was wrong. In the remainder of this paper, I propose to attempt to capture Marx's insight (which I believe to have been fundamentally correct) by means of an alternative analysis. My strategy will be to take what has now become the standard physical quantities reproduction model of an  $n$ -industry single product system (without land or fixed capital, for the sake of simplicity), and to introduce into it a formal peculiarity. After analyzing the structure of this altered model in general terms, I will explain why I think it is both faithful to Marx's insight and true to capitalism to identify the labor-producers (that is, the working class) as the formally distinguishable element in the system. I shall then do a bit of theoretical development by way of indicating what analytic insights can be gained from this new approach. Finally, I shall indicate how this fragment of theory fits into the larger project which I have undertaken, and point the way to theoretical steps which remain to be taken.

One lengthy aside before proceeding to the positive analysis. It might be objected that labor is, in fact, formally distinguished from all other inputs into the production process. The argument might go like this: many inputs are directly or indirectly required for the production of all commodities in the system under examination, but

only labor is *directly* required. To this argument (which has been urged upon me by a number of colleagues and students), I reply that this fact, if it is a fact, is not theoretically significant, because neither Marx nor Ricardo holds that natural prices are regulated by direct labor inputs alone. The quantity of labor directly required for the production of a unit of some commodity is not, by itself, a significant datum. Since all the labor inputs require food as input into their production, it follows that there will always be at least two equally suitable "substances of value." To this reply, it has been objected that labor is still formally distinguished by the fact that it, and it alone, is an input directly or indirectly required for the production of every output *in every logically possible capitalist system*. The point being made here is that no one food is a staple of the working class diet in every society. In some societies, rice plays that role; in others, potatoes; in still others, wheat, or maize. But in all capitalist economies, labor is directly or indirectly required for the production of all outputs. So, it is concluded, in the enlarged logical space of all possible capitalist economies, labor is formally distinguishable from all other inputs.

This argument too is faulty, in my judgment, but its flaw is rather more subtle. The problem is essentially one of classification and disaggregation. If we treat *food* as a single category for purposes of analysis, then it will of course be true that in all possible capitalist societies, both food and labor will be directly or indirectly required for all outputs. ("Possible" here is not meant to encompass science-fiction fantasies of societies in which workers can live without eating.) On the other hand, if we disaggregate food into rice, beef, maize, and so on, then no one subcategory will be directly or indirectly required for the production of all commodities in all possible capitalist societies. But if we disaggregate labor into tailoring, carpentering, tool-and-die-making, and so forth, then no one subcategory of labor will be directly or indirectly required for the production of all commodities in all possible capitalist societies. We might think to get around this difficulty by classifying labor into lowest-skill, median-skill, and high-skill. It will then be true, of course, that in all possible capitalist societies, lowest-skill labor will be required directly or indirectly for the production of all commodities (omitting, for the sake of simplicity, the possibility that in some society the lowest-skilled labor might

be employed only for the production of luxury goods and services). But the actual concrete physical activities classified as "lowest-skill labor" will vary considerably from society to society. We could as well classify food into subsistence food, medium-quality food, and luxury food. It would then be the case that subsistence food, like lowest-skill labor, was directly or indirectly required for the production of all commodities in all possible capitalist societies. But the actual physical foods classified as subsistence food would vary considerably from society to society.

I conclude that my analysis is correct, and that labor cannot be formally distinguished from all other inputs on the grounds either that it is the only input directly or indirectly required for production of all commodities in our economy; or that it is the only input so required in all possible capitalist economies.

#### IV. AN ALTERNATIVE ANALYSIS OF THE ROLE OF LABOR IN A CAPITALIST ECONOMY

Let us begin with some new notation and a new model. Thus far we have been examining a model in which labor is formally and notationally distinguished as an input not represented as the output of any industry; and in which the system of industries exhibits a uniform rate of return on the value of capital invested—a uniform profit rate. Let us now consider instead a perfectly general  $n$ -industry model, in which there are  $n$  single-product industries, and  $n$  inputs. For the time being, we shall in no way distinguish labor from any other input. The implication of this notation, of course, is that labor is a produced commodity which exchanges in the market at a natural price determined by the entire system of price equations. No assumptions are made about the organic composition of capital, and in fact that concept will play no role in our argument. The single major formal innovation will be the assumption that one industry, arbitrarily labeled the first, does *not* earn the going rate of return, but instead earns a rate of return different from that earned in the rest of the economy. Using the letter *rho* to represent that different rate of return, we can exhibit the structure of our economy in the following system of equations. Notice that we have a system of  $n$  equations in  $n + 2$  unknowns

(namely, the  $n$  prices,  $\pi$ , and  $\rho$ ). The number can be reduced to  $n + 1$  by the familiar device of adopting one price as numeraire, and since the choice is arbitrary, we shall select the price of the first commodity,  $p_1$ , as numeraire, and set it equal to 1. We now have this set of equations:

$$\begin{aligned}
 (13) \quad & [a_{11} + a_{12}p_2 + \dots + a_{1n}p_n] (1 + \rho) = 1 \\
 & [a_{21} + a_{22}p_2 + \dots + a_{2n}p_n] (1 + \pi) = p_2 \\
 & \quad \cdot \quad \quad \cdot \quad \quad \cdot \quad \quad \cdot \quad \quad \cdot \\
 & \quad \cdot \quad \quad \cdot \quad \quad \cdot \quad \quad \cdot \quad \quad \cdot \\
 & \quad \cdot \quad \quad \cdot \quad \quad \cdot \quad \quad \cdot \quad \quad \cdot \\
 & \quad \cdot \quad \quad \cdot \quad \quad \cdot \quad \quad \cdot \quad \quad \cdot \\
 & \quad \cdot \quad \quad \cdot \quad \quad \cdot \quad \quad \cdot \quad \quad \cdot \\
 & [a_{n1} + a_{n2}p_2 + \dots + a_{nn}p_n] (1 + \pi) = p_n
 \end{aligned}$$

where the  $a_{ij}$  are unit-input coefficients.

If we now adopt the following conventions:

$$a' = [a_{12}, a_{13}, \dots, a_{1n}]$$

$$A = [a_{ij}], i, j = 2, 3, \dots, n$$

$$p = \begin{bmatrix} p_2 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ p_n \end{bmatrix}, \text{ and } a = \begin{bmatrix} a_{21} \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ a_{n1} \end{bmatrix}$$

then this system can be expressed in matrix notation as:

$$(14) \quad [a_{11} + a'p] (1 + \rho) = 1$$

$$(15) \quad [a + Ap] (1 + \pi) = p$$

Solving (15) for  $p$ , we get:

$$a + Ap = \frac{1}{1 + \pi} p$$

$$a = \frac{1}{1 + \pi} p - Ap$$

$$a = \left[ \frac{1}{1 + \pi} I - A \right] p$$



Since the matrix  $[\frac{1}{1+\pi}I - A]$  has an inverse, we can solve for  $p$ , obtaining:

$$p = [\frac{1}{1+\pi}I - A]^{-1} a$$

Substituting this result into (14), we obtain:

$$\{a_{11} + a' [\frac{1}{1+\pi}I - A]^{-1} a\} (1 + \rho) = 1$$

The matrix  $a'$  is  $1 \times n - 1$ ;

$$[\frac{1}{1+\pi}I - A]^{-1} \text{ is } (n - 1) \times (n - 1)$$

$a$  is  $(n - 1) \times 1$ . Hence the expression  $[a' [\frac{1}{1+\pi}I - A]^{-1} a]$  is a scalar, and we can solve for  $\rho$ , obtaining as our result:

$$\rho = \frac{1}{a_{11} + a' [\frac{1}{1+\pi}I - A]^{-1} a} - 1$$

Examining this equation, we find that as  $\pi$  increases,  $\frac{1}{1+\pi}$  decreases. Therefore, by the Perron-Frobenius theorems, all the elements of  $[\frac{1}{1+\pi}I - A]^{-1}$  increase. It follows that the denominator as a whole increases, and therefore that  $\rho$  decreases. So, as we might have expected, *there is an inverse (but, of course, non-linear) relationship between  $\rho$  and  $\pi$ .*

We can also examine the extremal values for  $\rho$  and  $\pi$ . If  $\pi = 0$ , then equations (14) and (15) reduce to:

$$(16) \quad (a_{11} + a' p) (1 + \bar{\rho}) = 1 \text{ where } \bar{\rho} = \text{the maximum value of } \rho.$$

$$(17) \quad a + Ap = p$$

Solving equation (17) for the price vector,  $p$ , gives us:

$$\begin{aligned} a &= p - Ap \\ a &= [I - A] p, \text{ or} \\ (18) \quad p &= [I - A]^{-1} a \end{aligned}$$

But this is simply the vector of  $\mathbf{1}$ -values, which is to say the quantities of commodity  $\mathbf{1}$  directly and indirectly required per unit of each other commodity. So our first conclusion, not surprisingly, is that when  $\pi = 0$ , commodities  $2$  through  $n$  trade at their  $\mathbf{1}$ -values. [If commodity  $\mathbf{1}$  should turn out to be labor, then the other commodities trade at their labor values.]

Substituting (18) in equation (16) and solving for  $\bar{p}$ , we obtain:

$$(19) \quad \bar{p} = \frac{\mathbf{1}}{a_{11} + a' [I - A]^{-1} a} - \mathbf{1}$$

Consider the expression on the right-hand side of this equation. Using Marx's conventions, with the value of output equal to the value of constant capital,  $c$ , plus the value of variable capital,  $v$ , plus the value of surplus value,  $s$ , we observe that in this normalized system,  $c_i + v_i + s_i = \lambda_i$  for each industry  $i$ . Now, if we let commodity  $\mathbf{1}$  be labor, then  $a_{11}$  is the labor-input coefficient for the labor industry, so  $a_{11} = v_1 + s_1$ .

$[I - A]^{-1} a$  is the column vector of labor values of commodities  $2$  through  $n$ , and  $a'$  is the row vector of the non-labor inputs, per unit output, in the labor industry. Hence  $a' [I - A]^{-1} a$  is the labor value of the non-labor inputs in the labor industry, which is to say:

$$a' [I - A]^{-1} a = c_1$$

It follows from the price equation, (14), for the first industry that

$$(v_1 + s_1 + c_1) (\mathbf{1} + \bar{p}) = \mathbf{1}$$

$$\text{or } \lambda_1 (\mathbf{1} + \bar{p}) = \mathbf{1}$$

$$\text{Hence (20) } \bar{p} = \frac{\mathbf{1}}{\lambda_1} - \mathbf{1} = \frac{\mathbf{1} - \lambda_1}{\lambda_1}$$

So we see that when  $\pi = 0$ , the rate of return on capital invested in the labor industry,  $\bar{p}$ , equals what Marx defines as the exploitation rate,  $\sigma$ , which is to say the ratio of unpaid to paid labor.

We can analyze the situation further by looking at the effect on the  $i$ th industry of the fact that with  $\pi = 0$ , the positive  $\bar{p}$  drives up the price of the first commodity, labor. The equation for the  $i$ th industry is:

$$a_{i1} + a_{i2}p_2 + \dots + a_{in}p_n = p_i = \lambda_i$$

As we have seen,  $1 - \lambda_1 = (v_1 + s_1 + c_1)\bar{p}$

Hence,  $1 = \lambda_1 + (v_1 + s_1 + c_1)\bar{p}$

We can therefore rewrite the equation for the  $i$ th industry as:

$$a_{i1} [\lambda_1 + (v_1 + s_1 + c_1)\bar{p}] + c_i = \lambda_i$$

since  $c_i = a_{i2}p_2 + \dots + a_{in}p_n$

Or:  $a_{i1}\lambda_1 + (v_1 + s_1 + c_1)\bar{p} a_{i1} + c_i = \lambda_i$

The second term on the left, namely  $(v_1 + s_1 + c_1)\bar{p} a_{i1}$ , is simply  $s_i$ , which is to say the surplus value extracted from the labor inputs in the  $i$ th industry, per unit of output of the  $i$ th commodity. But because this is a model of an economy ruled by a market price system, the capitalists in the  $i$ th industry are forced to pay the market price for labor, which is 1 per unit. In this way, the surplus extracted from the labor inputs is distributed away to the labor producers.

If we introduce a vector of activity level multipliers,  $\beta_i$ , to define the actual magnitude of the system, then the total profit earned by the labor-producing industry equals the surplus labor value extracted from the labor inputs in all lines of production, as the following demonstration proves:

In the first industry, the total profit earned is given by the expression:

$$(21) \quad \beta_1 (a_{11} + a'p)\bar{p}$$

In each industry,  $i$ , for  $i = 2, 3, \dots, n$ , the labor input [or, more generally, the input of commodity 1, whatever that happens to be], costs:

$$\begin{aligned} \beta_i a_{i1} &= (\beta_i a_{i1}) \times 1 \\ &= (\beta_i a_{i1}) (a_{11} + a'p) (1 + \bar{p}) \quad [\text{See equation (16)}] \end{aligned}$$

The labor value of the labor input (or, more generally, the 1-value of commodity 1, whatever that happens to be) is given by the expression:

$$\beta_i a_{i1} (a_{11} + a'p) \text{ inasmuch as } p = \lambda \text{ when } \pi = 0. \quad [\text{See equation (18)}]$$

So, the surplus transferred, by market exchange, from industry  $i$  to industry 1, is:

$$\beta_i a_{i1} (a_{11} + a'p)\bar{p}$$

The total value transferred from industries 1 through  $n$  to industry 1, by market exchange, is, therefore, given by the expression:

$$(22) \quad \sum_{i=1}^n \beta_i a_{i1} (a_{11} + a'p)\bar{p} = (a_{11} + a'p)\bar{p} \sum_{i=1}^n \beta_i a_{i1}$$

[This sum *includes* the "profit" that is transferred, so to speak, from industry 1 to itself, insofar as industry 1 consumes commodity 1 as input. Proper bookkeeping requires that industry 1 charge itself the full price on its books.]

On the assumption that there is no final demand for commodity 1, so that the producers in industry 1 do not consume any of their own output unproductively, it follows that:

$$(23) \quad \sum_{i=1}^n \beta_i a_{i1} = \beta_1$$

Substituting (23) in (22), and comparing with (21) completes the proof.

At the opposite extreme,  $\rho = 0$  and  $\pi$  is at a maximum,  $= \bar{\pi}$ . In this case, we obtain:

$$\rho = 0 = \frac{1}{a_{11} + a' \left[ \frac{1}{1 + \bar{\pi}} I - A \right]^{-1} a} - 1$$

or:

$$1 = a_{11} + a' \left[ \frac{1}{1 + \bar{\pi}} I - A \right]^{-1} a$$

Consider what happens to  $\bar{\pi}$  as one or another of the terms of this equation varies. From the Perron-Frobenius theorems, we know that

$\left[ \frac{1}{1 + \bar{\pi}} I - A \right]^{-1}$  is a decreasing function of  $1/(1 + \bar{\pi})$ , and, therefore, an increasing function of  $\bar{\pi}$ . That is to say, every element of  $\left[ \frac{1}{1 + \bar{\pi}} I - A \right]^{-1}$  varies directly with  $\bar{\pi}$ .

Consider in turn the relationship between  $\pi$  and  $a_{11}$ ,  $a'$ , and  $a$ . These, it should be recalled, are respectively the labor input per unit of labor output, the non-labor inputs per unit of labor output (customarily called the real wage), and the labor inputs into the non-labor industries.

1.  $\bar{\pi}$  varies inversely with  $a_{11}$ . As  $a_{11}$  rises,  $(1 - a_{11})$  falls, hence  $[a' [\frac{1}{1 + \bar{\pi}}I - A]^{-1} a]$  falls, hence every element of  $[\frac{1}{1 + \bar{\pi}}I - A]^{-1}$  falls, hence  $\bar{\pi}$  falls. When  $a_{11} = 0$ ,  $\bar{\pi}$ —all else being constant—falls, hence  $\bar{\pi}$  falls. When  $a_{11} = 0$ ,  $\bar{\pi}$ —all else being constant—falls, hence  $\bar{\pi}$  falls.
2. As the real wage,  $a'$ , rises,  $\bar{\pi}$  falls (as we might expect). As the real wage,  $a'$ , falls,  $\bar{\pi}$  rises. Strictly speaking, this means that as any one or more of the components of  $a'$  rise, and none fall,  $\bar{\pi}$  must fall.
3.  $\bar{\pi}$  varies inversely with  $a$ . This simply indicates that as the amount of labor required by one or more processes rises, while the cost of producing labor itself remains unchanged, the profit rate will fall.

It is time to ask what economic meaning, if any, can be imputed to these calculations, and what light, if any, they can shed on our original problem, which was to explain the origin and nature of profit in a capitalist economy. We have been looking at the consequences that flow, in our model, from the imposition of some exogenous constraint on the ability of producers in one sector to earn the going rate of return on the value of capital invested. Since the mechanism that insures a uniform rate of return is the free flow of capital from sector to sector—capitalists shifting their capital out of sectors yielding a relatively low return and into sectors yielding a relatively high return—our exogenous constraint must consist of some impediment to the flow of capital into or out of the industry in question.

Now, I should like to suggest that this is precisely the proper way to construe labor in a capitalist economy. This is not, of course, the way in which labor is treated either by Ricardo and the neo-Ricardians or by the neo-classical school. Sraffa and his followers treat labor as a non-reproduced resource, the amount of which is given for a given

economic system. Indeed, the total quantity of labor available to the system is one of the normalizing constraints on the system, in Sraffa's analysis. But this is clearly *not* the way in which Marx thinks of the matter. Labor power is described by him as a *commodity*, and a commodity, technically speaking, is a good produced for the purpose of exchange on the market in such a manner as to be characterizable as having been produced by abstract, socially necessary labor. (What this means, among other things, is that there is a competitive market for the commodity that drives out inefficient means of production, and so on.) Like all other commodities, labor power has a natural price, toward which its market price tends. So we must render labor power, in a model of a capitalist economy, as a produced commodity, and insofar as we restrict ourselves to the simplification of single-product industries, we must identify one of the industries in our model as the labor industry—that is, the industry producing labor power for exchange on the market.

Precisely at this point, as Marx is at great pains to make clear, the whole superstructure of liberal bourgeois philosophy and political theory and law is introduced to justify the treatment of labor power as a commodity. Workers are treated in law, in ideology, and in philosophy as small producers, petty entrepreneurs who bring their product, like any other capitalists, to the market and exchange it for the products of other capitalists' enterprises. Their fixed capital is their bodies, which—according to classical liberal philosophy and jurisprudence—they own. Their circulating capital is the fund they spend for food and clothing. Assuming that they live at the level of bare subsistence, the worker-capitalists are not likely to hire labor services (although they may be forced to go to a doctor from time to time). Hence, all their capital will be constant capital, none of it will be variable capital, to use Marx's terminology.

Why are the workers unable to move their capital freely to sectors paying a higher rate of return? The simple answer, of course, is that their fixed capital is their bodies and their circulating capital is the food they eat to stay alive. A steel producer who finds the return in steel declining can, given a long enough period of time, cash in his investment and shift his capital to clothing, rental housing, or luxury appliances. The worker who notices the absence of any signif-

icant rate of return on her capital investment, and who, like any prudent capitalist, wishes to shift to a more profitable line, will find it necessary to separate herself ("alienate herself," to use the technical legal term) from her body. And by a quite unfortunate metaphysical accident—which, however, can scarcely be blamed on capitalism itself!—she is unable to survive that particular liquidation of her investment!

It will be objected that workers are not *really* petty capitalists. Just so. But the objection entirely misses the point of Marx's analysis. The workers must be made to *appear* as petty capitalists, in law, in political philosophy, and in the *formal theory of political economy*. A political economy that fails to model the essential mystification and ideological self-deception of capitalist economic, political, and legal relationships will be an inadequate theory of capitalism. An adequate political economy must *capture that feature as false*, in order to be true to the reality and to the appearance of capitalism.

We have here a very strange requirement indeed. We need a *formal model* of an ironic, dialectical relationship between appearance and reality. The trouble with other attempts to capture Marx's meaning is that they are either literary renderings, which preserve the irony and the intricate interrelation between appearance and reality, but without the formal structure that will allow us to calculate the magnitudes of the relevant variables; or else they are formal models, like the Sraffa model, that lose entirely the element of mystification and self-deception.

If we agree with Marx that capitalism has *its own mad logic*, then we will search for a model that embodies both the logic and the madness of capitalism. I suggest that the correct way to begin this process is to treat the workers as though they were petty entrepreneurs, producers, producing a commodity—labor power—for the market, and then capture the inner madness of this way of thinking of them by stipulating that they, alone among all capitalists, are unable to shift their capital about from sector to sector.

Under what circumstances will  $\rho$  actually go to zero? Presumably, the answer is: when a reserve of capitalists lurks at the edges of the market, possessed of a capital which they can use for no other purpose than to produce labor power, and willing therefore to throw it into production for any price that will enable them to halt its otherwise

inexorable depreciation. Which is to say, the reserve army of the unemployed.

We may gain some further insight into the capitalist system by asking the question: what precisely do the workers *lose* by their inability to earn any return on their capital investment in the production of labor? Consider the system of equations expressing the price system for this special situation, and compare it with the system of equations expressing the labor-values corresponding to the same system.

$$\begin{array}{ll} 1. \text{ The price system} & 2. \text{ The value system} \\ a_{11} + a' p = 1 & a_{11} + a' \lambda = \lambda_1 \\ [a + A p] (1 + \bar{\pi}) = p & a + A \lambda = \lambda \end{array} \quad \lambda = \begin{bmatrix} \lambda_2 \\ \dots \\ \lambda_n \end{bmatrix}$$

The solution to the price system, when substituted into the first equation, yields:

$$(24) \quad 1 = a_{11} + a' \left[ \frac{1}{1 + \bar{\pi}} I - A \right]^{-1} a$$

The solution to the associated value system, when substituted into the first equation yields:

$$(25) \quad \lambda_1 = a_{11} + a' [I - A]^{-1} a$$

Subtracting (25) from (24) gives us:

$$(26) \quad 1 - \lambda_1 = a' \left[ \left[ \frac{1}{1 + \bar{\pi}} I - A \right]^{-1} a - [I - A]^{-1} a \right]$$

If  $\bar{\pi} > 0$ , then  $\frac{1}{1 + \bar{\pi}} < 1$ , in which case

$$\left[ \frac{1}{1 + \bar{\pi}} I - A \right]^{-1} > [I - A]^{-1}$$

Consequently,

$$\left[ \frac{1}{1 + \bar{\pi}} I - A \right]^{-1} a > [I - A]^{-1} a$$

In words, each price in the price vector, when the wage is taken as numeraire and all prices are expressed in units of labor, is larger than the corresponding element of the value vector which expresses



the quantity of labor directly or indirectly required for the production of one unit of that commodity. The expression:

$$a' \left[ \left[ \frac{1}{1 + \bar{\pi}} I - A \right]^{-1} a - [I - A]^{-1} a \right]$$

gives the excess, in units of labor, of the *price* of the real wage,  $a'$ , over the *value* of the real wage. The equation:

$$(26) \quad 1 - \lambda_i = a' \left[ \left[ \frac{1}{1 + \bar{\pi}} I - A \right]^{-1} a - [I - A]^{-1} a \right]$$

states that the surplus labor extracted from each unit of labor input exactly equals the surplus value that the laborer must pay for the non-wage components of the real-wage basket.

In this way, we can see the result of the introduction of a uniform rate of return on capital invested in all sectors save the labor-producing sector. As a consequence of the introduction of the profit rate, all prices are driven up relative to labor. Workers are therefore required to pay more for the real-wage bundle than its labor value. The difference between price and value is redistributed by the market mechanism to the other  $(n - 1)$  sectors.

## V. CONCLUSION

Let us stand off a bit from the detail of the model and reflect on what our analysis has taught us. According to Marx, the central craziness (*Verrücktheit*, he calls it) of capitalism is the fact that the capacity to labor, to transform nature purposefully and artfully in the service of human need, is treated in the marketplace as a *commodity*. This absurdity has its historical roots in the separation of the working class from the means of production. It is, under capitalism, the root and source of exploitation, which, technically speaking, is the extraction from a factor of production of more value than is embodied or contained within it. Thus far, I follow Marx completely. His insight is, in my judgment, correct, as are the essential elements of his historical account. (The two other fundamental crazinesses of capitalism, to which Marx devotes equal attention, are the emergence of money and capital as objectively real social forms, and the existence

of internal crises of over-production—this essay is not concerned with either of those “contradictions,” although a fully adequate reconstruction of Marx’s political economy must deal with both in such a manner as to establish their relationship to the treatment of labor power as a commodity.)

Marx locates exploitation in the sphere of production, not in the sphere of circulation (behind the factory door, not in the sunlit market), and identifies exploitation with the extraction of surplus labor-time from the workers. His principal analytic maneuver is the distinction between labor and labor power, and his most powerful justification for the labor theory of value is its success, in conjunction with that distinction, in identifying the precise source and quantity of surplus value extracted by the capitalists in the process of production.

Marx’s analysis of exploitation is incorrect, as we have seen. But his central insight is perfectly correct: the root of exploitation, and the source of surplus value, is the treatment of labor power as a produced commodity. However, exploitation does *not* take place in the sphere of production; *nor* does it take place in the sphere of circulation. Rather, the extraction of the surplus from the workers takes place *in the interaction between the spheres of production and circulation*. To be precise, the extraction of the surplus comes about through the fact that the workers are forced to sell their product (labor power) at its labor value, but must purchase the non-labor inputs into their production process (that is, their food, clothing, and shelter) at prices driven above their values. Capitalists are able to earn the economy-wide rate of profit because they are able to shift their capital into or out of lines of production according to whether the short-term, or market profit-rate is above or below the natural or economy-wide profit rate. The anomalous status of workers prohibits them from shifting their “capital” about in search of a higher rate of return, and the existence of a reserve army of the unemployed effectively drives the rate of return in the labor-producing industry down to zero.

In *Capital*, Marx represents the workers, with bitter irony, as suffering exploitation because of the sheer metaphysical accident that their product happens to be capable of creating exchange value

when it is consumed as a use value. In short, Marx says that the workers can be exploited *because* labor is the substance of value. The truth, not surprisingly, is the exact opposite: labor is the substance of value *because* the workers can be exploited! To put the same point somewhat differently, the distinguishing logical feature of labor in a formal model of a capitalist economy is *not* that it must be chosen as numeraire, for that is simply false; nor that commodities, at their natural prices, exchange in proportion to the quantities of labor directly or indirectly required for their production, for that too is false. The distinguishing logical feature of labor in a capitalist economy is that the industry producing it does not in general earn the uniform rate of return on the value of capital invested. Any notational system which contains within it enough in the way of formal differentiation to permit an adequate representation of the formal structure of capitalism will preserve this logical peculiarity. It makes no difference whether we use the Greek letter lambda to signify that we are representing labor. What matters is that the logical, or formal, relationships between labor and the other elements of a capitalist economy be modeled in our formal system. So long as this condition is met, the formal structure we set out will be adequate to serve as the basis for an analysis of exploitation consonant with Marx's central insights.

The point being made here is quite general, and when this model is extended, in subsequent developments of this analysis, to take account of such theoretical complications as land, fixed capital (and joint production), and—most difficult of all—the emergence of money as something more than, and different from, mere commodity-money, it will be necessary to introduce new formal elements into the structure corresponding to the more elaborate formal structure of the reality being analyzed.

Finally, let us explore some of the implications and ramifications of the analysis presented thus far.

We can gain an interesting perspective on the history of the struggle over the development of labor unions by analyzing it in terms of the notion of labor power as a produced commodity. In the United States, labor unions were viewed by the state and by the courts as combinations in restraint of trade, and attempts to organize were fought by

appeal to anti-monopoly laws. This tactic clearly exhibits the anomalous and ideologically mystified classification of labor power as a commodity. The workers are treated as producers of a commodity, labor power. As such, they are conceived as earning the going rate of profit under competitive conditions. Combinations in restraint of trade can only drive their return above the going profit rate, with the result that non-optimal production techniques will be adopted, and less than a maximally efficient quantity of their commodity will be produced and sold. But as our analysis shows (and as any worker knows), the reality is that labor producers—that is, workers—are barred by the special conditions of their line of business from earning any return at all on their “capital investment.” *Their* combination in unions is designed to drive up the wage so that, in our model,  $\rho$  takes on a positive value. Because  $\rho$  and  $\pi$  are inversely related, this, of course, lowers the profit rate. But even according to the established theory of capitalist economics itself, non-optimal allocation of resources would result only when  $\rho$  rose above  $\pi$ . For any value of  $\rho < \pi$ , the system could be described as a sub-optimal, with industries 2 through  $n$  engaged, as it were, in a collusive effort to hold down the rate of return in industry 1. The ideological misrepresentation of unionization as an economically distorting mode of collusion continues to the present day, and plays a significant role in conservative attacks on the labor movement. *Notice*, by the way, that this view is shared by liberal pluralist defenses of unionization, which appeal to interest-group political theory but do not deny the economic argument in purely economic terms. [Presumably, excessively labor-intensive techniques will be favored by this.]

In the simplest form of our model, with no fixed capital (and no joint production), the inputs into the labor-power industry consist entirely of circulating capital consumed in one cycle of production. Assuming that the input coefficients are maintained at subsistence level by the presence of the unemployed, labor producers will have *no* options in choice of technique or allocation of a depreciation fund. Now, when we make the model more complex, two degrees of indeterminacy enter, as it were. First, we may introduce a factor for depreciation throughout the model. In the labor-producing industry, this factor, as we have suggested, serves to enable the workers to

raise children and thereby replace their bodies, as they wear out, with offspring. Second, collective bargaining, labor shortages, political action leading to minimum wage laws, and so on, may drive  $\rho$  up to some positive value, thereby producing a value surplus, or profit, in the labor-producing industry. The indeterminacy thus introduced concerns the disposition of either the depreciation fund or the profit. Let us consider each separately.

### *The Depreciation Fund in the Labor-Producing Industry*

Once we allow for fixed capital and depreciation, all manner of theoretical complexities are introduced. Leaving aside the formal analysis of fixed capital (in terms, presumably, of joint production, and so on), let us concentrate on the role of fixed capital and depreciation in the labor-producing industry. In order to simplify matters, let us for the moment ignore such elements of fixed capital as privately owned housing, furnishings and cooking implements, clothing, and so on, and concentrate instead on two elements alone: the worker's body itself, and the marketable skills he or she possesses. Workers have a number of options with regard to the allocation of the depreciation fund, and the economic system is thus correspondingly indeterminate to a degree. Later on, we shall wish to investigate the various endogenous and exogenous factors that work to constrain the choices of the workers and reduce the degree of indeterminacy. Workers can:

1. Undertake to replace the body as it wears out by conceiving and rearing children.
2. Increase the productivity of their offspring by restricting their number and raising their skill levels (through apprenticeships, education, and so on, which may involve holding them off the labor market for a period).
3. Restrict or entirely reject reproduction and use some or all of the depreciation fund to raise *their own* skill levels, through retraining, education, and so on.
4. Accumulate the depreciation fund and invest it in some other line of production which *does* earn the going rate of profit. In

this way, a portion of the wage may genuinely be converted into capital—save that pressure from the reserve army of the unemployed may drive down the wage until it suffices only for reproduction and not for depreciation.

5. Spend the depreciation fund on “luxuries”—that is, raise their standard of living.
6. Or some combination of the above.

How shall we treat these various possibilities? First of all, at the level of surface appearances, we will observe worker behavior that can be interpreted—*and may be interpreted by the workers themselves*—as the straightforward rational selection of one of these options. So one may find a theory of human capital developing in economics that interprets consumption patterns, wage rates, and so forth along these lines. And one may observe behavior that seems to confirm these theoretical explanations.

What is the reality that is mystified in this situation? I suggest there are *three* elements of mystification, of quite different sorts:

1. Labor power is still construed as a produced commodity, even though its producers are uniquely constrained by their inability to shift more than marginal amounts of their capital to other lines of investment.
2. The role of the *family* in the private sphere is totally ignored, with the result that workers are systematically exploited *above and beyond* the exploitation resulting from the zero or low  $\rho$ . The existence of the family makes it possible to pay women and children below-subsistence wages, because their wages are supplementary to family income, rather than the principal source of subsistence. In this way, the wages of principal wage-earners are held down, so that a greater amount of labor is extracted from the family unit for a given total wage. The interdependence of the available wages for men, women, and children is obscured from view by the apparent atomic independence of each wage-earner in the marketplace, and by the economic rationalization that workers, in competitive circumstances, earn a wage equal to the marginal product of their labor.
3. Most important, perhaps, the political pattern of domination

in the work-place is obscured by the mythology of the wage-bargain as commodity exchange. The "accident" that the labor-power producers must accompany their product to the place of productive consumption is in fact the essential and central fact of capitalist production, a fact *unique* to the labor-power producing industry. The capitalist's political domination of the worker in the production process is the key to the extraction of a surplus from the workers. It is also, paradoxically, the key to the downfall of capitalism, for if the labor power could be separated from its producer, then there would be no tendency for the workers to unite in opposition to capital. This is the familiar contradiction in capitalist development sketched by Marx and Engels in the well-known passage in the *Manifesto*. [Cf. Braverman on all this.]

### *The Disposition of a Profit in the Labor-Producing Industry*

When collective bargaining, labor shortages, skill shortages, or political action drive up  $\rho$  to a positive magnitude, then a profit is earned by the labor-producers. Obviously, this profit may be disposed of in the variety of ways described under heading A. Consider three further points:

1. If some or all of the profit is consumed unproductively, then quite likely consumption patterns will change. It is unlikely, for example, that workers will spend part of their income for the same old subsistence wage-goods basket, and then spend the rest for "luxuries." Instead, they will alter their entire consumption pattern—white bread for black, butter for lard, decent housing for slum housing, and so on. This *can* be treated as the adoption of a different and deliberately sub-optimal technique of production—a theoretical move that will very nicely capture the anomalous status of labor-power production, for in no other industry will entrepreneurs have any rational reason for deliberately adopting a sub-optimal technique of production.

2. If workers choose to accumulate their profit through abstinence and thrift, they may then decide to invest in a small, self-owned and

operated business—such as a gas station, fast-food outlet, or candy store. What they will *gain* is escape from the political domination of industrial or corporate work. What they will pay for this escape, as J. K. Galbraith clearly shows, is self-exploitation in the form of extremely long hours, low hourly pay, and a small or negligible return on capital invested. This “American Dream” actually serves to extract enormous quantities of underpaid labor from the American labor force, thereby swelling the profits of the “planning sector” of major corporations.

3. As wages rise above subsistence, the possibility develops for redistributive taxation, and with it, the exacerbation of the split between the upper and lower working-classes, whose relationship to the work world is quite different. It is actually in the interest of capital to have an inner core of relatively skilled workers with seniority, security, and above-subsistence wages, and a floating mass of marginal workers whose repeated inclusion in, and then exclusion from, the work world is useful to capital.

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